

AND ANSWERS

TO THE

QUESTIONS

OF THE

STUDENT

OF THE



Ed. T. Seaton



HINTS AND ANSWERS
TO THE EXERCISES
IN
ELEMENTS OF ALGEBRA.

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ANSWERS.

EXERCISE XIV [b]. (PAGE 32.)

1. $6x^2 - 12x^2y + 7xy^2 - 3y^3 - y^2 + 2xy^3.$
2. $-8(m+n) + 5(a+b).$ 3. 0. 4. $8(m+n)^2 - y.$
5. $x^3 + x^2y + 7xy^2 + 2y^2 + y^3.$ 6. $\frac{3}{2}a^2 - a^3 - 2a^2b + \frac{1}{4}ab^2 + b^3.$
7. $10x \div y - 13m \div n.$

EXERCISE XV.

1. $(3 + 6b + 7a)x + (-2 - 4)y + m + n.$
2. $(a + m + 1)x + (1 + n - d)y.$
3. $(6a - 3b - 2)x + (1 + \frac{3}{4}b + \frac{3}{2}a)y.$
4. $(2d - 2f)x + (3e - 3d)y + (4f + 4e)z.$
5. $(a + b - 4)x + (\frac{1}{2}a^2 + c^2 - \frac{2}{3}a^3 - 6)y.$ 6. $3ax - 3by.$
7. $(a - 6)x + (5m + 5)\sqrt{y} + (b - 1)y - 3\sqrt{x}.$
8. $(a - c)x^2 + (b - a)y^2 + (c - b)z^2 + ax + by + cz.$
9. $(1 - 9a - 2b - c)x^n + (1 + 7b + 10a - 3abc)y^n.$

EXERCISE XVI [b]. (PAGE 34.)

1. $-9x^4 + 12x^3y + 6x^2y^2 - 18xy^3 + 21y^4 + 20.$
2. $p^2 - 17q^2 - 22r^2 + 17pq - 8yz^2 + 99.$
3. $-3(x - y) + 34(x - z).$ 4. $19(a - b) - 8(a + b) + 14a + b.$
5. $12\frac{x}{y} - 11\frac{y}{x} - 8\frac{z}{x} - 8\frac{a}{b} - 6\frac{y}{z}.$
6. $2x^2 - \frac{4}{3}xy - \frac{1}{2}y^2.$ 7. $-\frac{1}{8}a^3 - \frac{2}{3}a^2x - \frac{3}{2}ax^2.$
8. $-2x^3 + 3xy^2 - y^3 - 14x^2 + 2xy - 10y^2 + 2.$
9. $a^2b^2 - 3a^2bc - 3ab^2c - a^2c^2 - abc^2 - b^2c^2.$ 10. $a^4 - 2a^2b^2 + b^4.$
11. $(a^2 - b^2)x^2 + (b^2 + c^2)y^2 + (c^2 - a^2)z^2.$
12. $19a^m - 17b^n - 2c^p + 10d^q.$
13. $(a - p)x^3 + (q - b)x^2 + (1 - r)x + 1.$

14. $a^3 + a^2b + 6ab^2 - 2b^3 + 3b^2$. 15. $\frac{1}{2}y - \frac{1}{6}a - \frac{2}{3}x$.
 16. $-\frac{2}{3}(xyz - bx + cy)^2 - 4\frac{3}{10}(z - y + ax)$.
 17. Sum is $37\frac{3}{4}a - 12\frac{3}{4}b - 41\frac{1}{4}c - 7\frac{1}{4}d - 28\frac{1}{2}e$; the several remainders are $35\frac{1}{4}a - 9\frac{7}{4}b - 11\frac{1}{4}c - 1\frac{1}{4}d - 24\frac{1}{2}e$;
 $33\frac{1}{2}a - 11\frac{5}{8}b - 5\frac{3}{4}c - 6\frac{7}{8}d - 20\frac{1}{2}e$;
 $31\frac{3}{8}a - 12\frac{7}{8}b - 3\frac{5}{4}c - 5\frac{1}{4}d - 22\frac{1}{2}e$;
 $29\frac{1}{4}a - 8\frac{1}{2}b + 3\frac{1}{8}c + 1\frac{7}{8}d - 17e$;
 $22\frac{5}{8}a - \frac{3}{8}b - 4\frac{1}{4}c - 4\frac{1}{4}d - 12\frac{3}{4}e$;
 $16\frac{3}{4}a + 2\frac{7}{8}b - 1\frac{1}{4}c - 1\frac{1}{8}d - 7\frac{1}{2}e$;
 $9\frac{1}{4}a + 8\frac{1}{2}b + 4\frac{3}{4}c + 6\frac{3}{4}d - 4\frac{1}{2}e$; 0.
 18. i. 0; a ; $\sqrt[3]{(3a^3)}$. ii. $26a^6$; $a + x$.

EXERCISE XVII [b]. (PAGE 39.)

- | | | |
|-------------------|--|-----------------------|
| 1. 6, 18. | 2. 30, 40. | 3. 35, 65. |
| 4. 4, 2, 10. | 5. 8, 40, 12. | 6. 500. |
| 7. 120, 137, 163. | 8. 120. | 9. 30 minutes. |
| 10. \$840. | 11. 7, 42. | 12. \$32, \$36, \$44. |
| 13. 24. | 14. \$138\frac{3}{4}, \$236\frac{1}{4}. | 15. £1200. |
| 16. 7 months. | 17. 450, 180, 140. | 18. 300. |
| 19. \$100. | 20. $12\{22x - \frac{5}{11}(20 - 33x)\} = 44x, x = \frac{3}{11}$. | |

EXERCISE XVIII. (PAGE 41.)

1. $a^2 + b^2 - c^2 - d^2$; $a^2 - b^2 + c^2 + d^2$. 2. $a^2 - 3b^2 + c^2$.
 3. $2m - n + 6$. 4. $-2x - 3y - 2z$. 5. $1\frac{1}{2}x - 4\frac{1}{2}y + 1\frac{1}{2}z$.

EXERCISE XIX. (PAGE 43.)

- | | | |
|--------------------------------|-----------------------|---------------------------|
| 1. $-2a + 3x + 3b$. | 2. $a + b + c$. | 3. $2ab + 4b^2$. |
| 4. $-3x - y + 4z$. | 5. $5 - 4x$. | 6. $2a - 3b - 3c + 4d$. |
| 7. $-4a$. | 8. $-x - 10y + 2z$. | 9. $-2x + 2y$. |
| 10. $2x - 6y - my + 4ab - 5$. | 11. $3a - 5b - c$. | |
| 12. 0. | 13. $-\frac{1}{3}y$. | 14. $\frac{1}{3}a - 2b$. |
| | 15. $\frac{1}{2}x$. | 16. 9. |

EXERCISE XX. (PAGE 44.)

1. i. $x - (a + b)$; $x - (a + 3b - 2y)$.
 ii. $x - (2m - 2n)$; $x - (3b - 2c - 5d)$.
 iii. $x - (2m + 3a - 2b)$; $x - (b - a - c - m + n)$.
 iv. $x - (a + b - c - 12)$; $x - \{(a + b) + (p + q) + (m - n)\}$.

2. i. $(2a - 4b - 3c)x - (6a + 3c)y + (4b - ac)z$.
 ii. $(a - b + c)x - (a + b - c)y - (a - b - c)z$.
 iii. $(12a - 15c)x - (12a + 4b + 6c)y - (12b + 3c)z$.
3. i. $2 + (7 - 2c)x + (5a - 3)x^2 + (9a - 7)x^3$.
 ii. $(2c - a^2)x^5 + (a - 3b)x^4 + (1 - m)x^3 + (4c - 3ab)x$.
 iii. $(1 - a)x^4 + (1 - b + c)x^3 + (b - 1)x^2 + (a - 7)x + 2$.
4. i. $-(3c^2 - 5a)x - (abc - 7)x^3 - (ab - 7)x^5$.
 ii. $1 - (a - 1)x - (1 - b)x^2 - (a - c + 1)x^3 - (a - b - 1)x^4$.
 iii. $-(a - 3b^2)x^4 - (1 - c)x^3 - (1 + 5c^2)x^2 - (b + c)x$.
5. i. $(a - c + 1)x^3 - (a + 2b + 1)x^2 + (b + c)x + 3$.
 ii. $(5a + 4c)x^3 + (7c - 6b + 3a)x^2 + (2a - 7b)x$.
 iii. $(a - b + c)x^2 - 2(a + b + c)x + ab - bc - ca$.
6. i. 6; 6. ii. -17; -9. iii. -1; -56.
7. $(a + b + c)(x + y + z)$.
8. $-3a - rx - (2 - b)x^2 + (4a - p - 1)x^3 + 2x^4$.
9. $(6y + 1)x^5 - (z + 2y)x^4 - (2z + 2)x$.

EXERCISE XXI [b]. (PAGE 48.)

1. 36; -48; 5; 9; -168; -180.
2. i. $m^3xyz; abcx^3; -24a^3b^3$. ii. $-36a^6m^4; -a^3b^3c^3x^2y^2z^3$.
 iii. $-14a^2b^3x^2; -18x^3y^2z^4; -5x^3y^3z^2$.
3. i. 40; -63; -2; -37. ii. 130; -880; 0. iii. $\frac{1}{2}; 29$.

EXERCISE XXII [b]. (PAGE 49.)

1. $a^3b^2c - ab^4 + abc^3; -\frac{5}{2}x^2 + \frac{5}{3}xy + \frac{1}{3}x^3$.
2. $3abxy + 6acxz + 15ax; 9x^4yz^2 - 12x^2y^3z^2 + 15x^2yz^4$.
3. $-15x^5y - 10x^4y^2 + 35x^3y^3 - 5x^2y^4; 3a^4 + 2a^5b - a^6b^2$.
4. $3x^3y^2z - 3x^2y^4z + 3x^2y^2z^4 - 12x^4y^4z^3; \frac{1}{4}a^2x - \frac{1}{16}abx - \frac{3}{8}acx$.
5. $-2a^6x^3 + \frac{7}{2}a^4x^4 + a^6x^4; -x^5y^5 + \frac{1}{16}x^6y^8$.
6. $\frac{3}{4}x^4y^2z^2 - \frac{3}{2}x^2y^3z^2 + \frac{3}{4}x^2y^4z^2 - x^3y^3z^3; \frac{5}{2}a^4x^2 - \frac{5}{2}a^3x^3 + a^2x^4$.
7. $-2(a + b)^4 + 2(a + b)^2; -3(a - b)^3 - 2(a - b)^5$.
8. $(m^2 - n)^7 + (m^2 - n)^5; 3(a + b)^{n+1} + 2(a + b)^{n+4}$.
9. $(a + b)^{n+1} + (a + b)^{n+1}; (a - b)^{n+1} - (a - b)^{n+2}$.

EXERCISE XXIII. (PAGE 50.)

1. $6x^2 - 13xy + 6y^2$.
2. $15x^3 - 3x^2y - 5bx + by$.
3. $x^3 - 9a^2x$.
4. $-10b^4 - 15ab^3 + 14ab^2 + 21a^2b$.
5. $a^3 + b^3$.
6. $a^3 - b^3$.
7. $a^6 - b^6$.
8. $y^5 - 5y^3 + 2y^2 + 6y - 4$.
9. $a^3 + \frac{1}{3}a^2b + \frac{1}{3}ab - 2ab^2 - \frac{1}{3}b^3$.
10. $(a^2 - b^2)x^{n+1}$.
11. $x^5 - a^5$.
12. $1 - x^6$.
13. $y^6 + 2y^4 - 7y^2 - 16$.
14. $\frac{1}{4}x^4 + y^4$.
15. $am + (an - bm)x + (ap - bn)x^2 - bpx^3$.
16. $a - (a^2 - b)x + cx^2 - (ac - b^2)x^3 + bcx^4$.
17. $x^8 - 3x^7 - 3x^6 + 6x^5 + 4x^4 - 5x^3 + 6x^2 - 12x + 6$.
18. $a^3 + b^3 - c^3 + 3abc$.
19. $x^3 + y^3 + 3xy - 1$.
20. $18x^8 + 27x^7 + 7x^6 + 3x^5 - 2x^4 + 65x^3 + 115x^2 + 49x + 6$.
21. $(x + y)^2 - (z + a)^2$.
22. $16a^2 + 24ab + 9b^2 - 4c^2 - 4cd - d^2$.
23. $16a^2 - 24ab + 9b^2 - 4c^2 + 4cd - d^2$.
24. $x^4 + 2x^3 + x^2 - y^4 + 2y^3 - y^2$.
25. $a^3 + 8b^3 - 27c^3 + 18abc$.
26. $81x^4 - 256a^4$.
27. $x^6 + 2x^3y^3 + y^6$.
28. $\frac{3}{8}x^4 - \frac{3}{2}ax^3 + \frac{1}{2}a^2x^2 - \frac{3}{8}a^4$.
29. $x^8 - a^8$.
30. $a^2x^{m+2} + abx^{m+2} + abx^{m+3} + b^2x^{m+3} - bx^3 - ax^2$.
31. $x^{2m} + x^{2m}y^m - x^my^m - y^{3m}$.
32. $x^8 - a^8b^8$.
33. $x^{16} - 1$.
34. $x^8 + x^4a^4 + a^8$.
35. $x^4 - y^4 - 4y^3 - 6y^2 - 4y - 1$.

EXERCISE XXIV [a]. (PAGE 53.)

1. $x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$.
2. $x^6 - 3x^4 + 3x^2 - 1$.
3. $2x^5 - 18x^4 + 39x^3 - 25x^2 + x + 1$.
4. $4x^6 - 5x^5 + 8x^4 - 10x^3 - 8x^2 - 5x - 4$.
5. $21x^8 + 14x^7 - 49x^6 - 8x^5 - 10x^4 + 41x^3 - x^2 - 14x + 2$.
6. $3x^6 + 7x^5 - 12x^4 + 2x^3 - 3x^2 + 13x - 6$.
7. $4x^6 - 8x^5 + 4x^4 - 12x^3 + 12x^2 - 6x + 9$.
8. $x^8 - 3x^6 + 6x^4 - 7x^2 + 3$.
9. $x^6 - 57x^4 + 266x^2 - 1$.
10. $18x^8 + 21x^7 + 8x^6 + x^5 + 63x^3 + 96x^2 + 43x + 6$.
11. $x^6 - 3x^4a^2 + 3x^2a^4 - a^6$.
12. $1 - x^7$.
13. $4 - 12a + 5a^2 + 14a^3 - 11a^4 - 4a^5 + 4a^6$.
14. $1 + x + x^3 + x^4 + x^{17}$.

50.)

$$5bx + by.$$

$$+ 14ab^2 + 21a^2b.$$

$$7. a^9 - b^9.$$

$$b - 2ab^2 - \frac{1}{3}b^3.$$

$$12. 1 - x^6.$$

$$15. akx^5 + (al + bk)x^4 + (am + bl + ck)x^3 + (an + bm + cl)x^2 \\ (bn + cm)x + cn.$$

[b.]

$$2. 0.$$

$$3. y^4 - 7y^2 + 10.$$

$$5. 729x^6 - 117649.$$

$$6. 2a^2 - 2ap - 2a^2n + p^2 + 2anp - 2an + np + 2an^2.$$

$$7. 0, \text{ put } a = b + c.$$

EXERCISE XXV. (PAGE 56.)

$$1. 3x; 7x; -3x^2; -5x^2.$$

$$2. -a^3c^4; a^4; -7a^2b^2c^2.$$

$$3. 3a^2; -2x; -2x^{m-3}.$$

$$4. 3a^3b^3c; \frac{1}{4}xy; -\frac{1}{2}x.$$

$$5. -a^2c; -a^{p-1}; ax^4.$$

$$6. -2a^{m-1}b^{n-1}; -a; ma; -2x^{2p+2}.$$

$$7. 3a^{m-n}p^{n-1}; 4a^3(x-y)^{n-2}; -(a+b)^{n-m}.$$

$$8. -4mx^4 \div 5a^3; -3ab \div 4c; 12a \div c.$$

$$9. a^2c^2 \div b; -a^{m-1}; x^{n-4} \div y^{n-4}. \quad 10. mx^m \div ny^n; a^2b^2c^2 \div x^3.$$

$$-27c^5 + 18abc.$$

$$y^3 + y^6.$$

EXERCISE XXVI. (PAGE 57.)

$$1. x - 2y; -x^2 + y^2; a^2b - a.$$

$$2. 1 - 3ax - 4a^2x^2; -1 \div x + 2abx.$$

$$3. -a + b + c; -a + b + b^2; \frac{1}{4}xy - \frac{1}{8}.$$

$$4. -3mx^{m-n} + 2am^3 - \frac{7}{4}a^4mx^{p-n}.$$

$$5. (a+b); 4(a-b)^2; a^{n-3} + a^{n-4}.$$

$$6. -\frac{1}{3}x^3y^3 + \frac{8}{3}x^2y - 2y; 3x^2 - \frac{3}{2}y + 4.$$

$$7. -a^n - x; a^{2n} - a^n x^n + x^{2n}. \quad 8. 6a^2xy - 5ay^2 + 3a^2xy - 4ay^2.$$

$$9. -\frac{1}{2}x^4 \div y + \frac{1}{4}x^3 - \frac{3}{4}x^2y^2 + \frac{1}{4}x^2.$$

$$10. \frac{2}{5c^2} - \frac{3}{5a^2} + \frac{4}{5a^2}; 4x^6 - x^2 + \frac{3}{2x^2}.$$

$$11. -\frac{x}{4y} + \frac{1}{4} - \frac{3}{4y}; \frac{a^3}{x^3} - \frac{a^3}{x^2} + \frac{a}{x} - 1.$$

$$12. \frac{3x^2}{2a^2} - \frac{5}{2a} + 3 + \frac{a}{2x^2}; (a-b)^{n-1}.$$

$$13. \frac{3}{2}(a+b)^3 - (a+b) + \frac{1}{2}.$$

$$14. a^{m-n} - (a-b)^{m-n}.$$

$$15. 2(x+y)^{m-2}(x-y)^{n-2} - (x+y)^{p-2}(x-y)^{q-2}.$$

$$16. (a+b)^{m-2}(a-b)^{n-2} - (a+b)^{m-3}(a-b)^{m-3}.$$

53.)

$$x^4 + 3x^2 - 1.$$

$$-14x + 2.$$

$$66x^2 - 1.$$

6.

EXERCISE XXVII. (PAGE 60.)

1. $x + 7$.
2. $a - 6$.
3. $3x - 2$.
4. $a - 24$.
5. $3x + 1$.
6. $3x - 7$.
7. $3x + 2$.
8. $4x + 3$.
9. $3x - 2y$.
10. $x - 7$.
11. $3x + 4$.
12. $5x - 1$.
13. $x^2 - y^2$.
14. $9x^2 + 4y^2$.
15. $8x + 3y$.
16. $x^2 + 14x$.
17. $4x^2 - x$.
18. $x^2 + 3x + 1$.
19. $a^2 + a - 1$.
20. $a^2x^2 + ax + 1$.
21. $2ab$.
22. $a^4 - 3ba^2 + 2b^2a$.
23. $x^2 + 2xy + 2y^2$.
24. $x^2 - 5x + 6$.
25. $x^2 - 2x + 2$; $-100x$.
26. $24x^2 - 2ax - 35a^2$.
27. $x^5 + xy^6 + 7ax$.
28. $-5a^2 + 4bd - 8cf$.
29. $3a^2 - 5b^2 + 3c^2$.
30. $3x^2 - x + 2$.
31. $a - b - c$.
32. $5a^2 + 3x^2$.
33. $p^2q + 4pq^2 + 2q^3$.
34. $x^2 - mx + m^2 - n$; $(m^2 - m^3)x$.
35. $x + y$; $y^{m+1} + 2xy^m$.
36. $x^{2n} - 2x^ny^n + y^{2n}$.
37. $ax^2 - bx^2 - a^2x + abx + a^3 - a^2b$. (Read a^3b^2 for a^2b^2 in text.)

EXERCISE XXVIII. (PAGE 61.)

1. $a^2 + b^2 + c^2 - ab + bc + ca$.
2. $x^2 - (a + b)x + ab$.
3. $y^4 - (m - 1)y^3 - (m - n - 1)y^2 - (m - 1)y + 1$.
4. $p^2 + q^2 + r^2 + pq + qr - rp$.
5. $1 - x + 2y + x^2 + 2xy + 4y^2$, $1 + x - 2y + x^2 + 2xy + 4y^2$.
6. $x^2 + y^2 + z^2 + 1$.
7. $3x^2 - x - 2$, rem $2x + 1$.
8. $2x^2 - x + 1$.

EXERCISE XXIX. (PAGE 64.)

1. $2x^3 + 3x^2 + 4x + 7$.
2. $x^2 - 2x + 4$.
3. $5x^2 - 10x + 2$, $3x^2 - 10x + 1$.
4. $x^3 - 3x^2 + 3x - 1$.
5. $4x^3 - 3x^2 + 2x + 2$.
6. $5x^2 - 12x + 12$, $12x - 72$.
7. $5x^2 + 10x + 5$, $-5x^2 - 10x + 27$.
8. $10x^3$, $10x^4 - 100$.
9. $1 - 2x + 3x^2 - 4x^3 + 5x^4$; $a^4 + 2a^3 + 3a^2 + 4a + 5$.
10. $x^2 + 2xy + 3y^2$; $m^2 - 2m + 3$.
11. $x^4 + 2x^3 + 3x^2 + 2x + 1$; $a^4 - 2a^3b + 3a^2b^2 - 2ab^3 + b^4$.
12. $3x^4 - 2x^3 - 2x + 3$.
13. $x^4 - 3x^2 - 4x + 15$, $54x^2 - 56x + 27$.
14. $x^5 - 3x^4 - 2x^3 + 2x^2 + 3x - 1$, $5x$.
15. $2x^3 - x^2 - 2x + 4$, $24x^2 - 12x + 10$.
16. $x^4 + 4x^3 + 6x^2 + 9x - 4$, rem. 5.
17. $2x^2 - 4x + 3$.

18. $x^8 - 2x^6 + 3x^4 - 2x^2 + 1$; $x^4 - x^3 + x^2 - x + 1$.
 19. $x^2 + (a+b)x + ab$; $x^2 - ax + bx - ab$. 20. $2y^2 - ay - \frac{1}{2}x$.
 21. $5x^4 - 24x^3 + 99x^2 - 400x + 1601$, rem. -6400 .
 22. $x^6 - x^4 - x^3 + x^2 + 2x + 1$, rem. 101.
 23. $2x^4 - x^3 + 2x^2 - 3x + 1$, rem. 10.
 24. $\frac{1}{3}x^5 + \frac{4}{3}x^4 - x^3 + \frac{1}{3}x^2 + \frac{2}{3}$, rem. $-3x^3 + 21x^2 - 3x + 14$; take factor 3 out of divisor and divide resulting quotient by 3.

EXERCISE XXX. (PAGE 69.)

1. 8. 2. 1. 3. $4w - 18$. 4. 3. 5. 2. 6. 1.
 7. 2. 8. 4. 9. 12. 10. $(a^2 + ab + b^2) \div 2(a + 2)$.
 11. $(n - b + a^2) \div (a - m + 2ab + c)$.
 12. $(bc - ab) \div \{a + c + b^2 + (a - c)^2\}$.
 13. $(a^2 + b^2 + ab) \div (a + b)$. 14. $6c \div (30 - 11a + 3b + 2c)$.
 15. $x = \frac{1}{2}(a + b)$; write P for $x - a$, and Q for $x - b$, and equation becomes $P^3 \div Q^3 = \{P - (a - b)\} \div \{Q + (a - b)\}$, and on clearing of fractions $P^2 - Q^2$ will prove to be a factor; $\therefore P^2 - Q^2 = 0$, $P + Q = 0$, etc. Or, multiply out.
 16. $(c^2 - ab) \div (a + b - 2c)$;
 equation is $(x + a) \div (x + b) = (2x + a + c) \div (2x + b + c)$;
 complete the divisions, square and transpose;
 $\therefore (c - b) \div (x + b) = (a - b) \div (2x + b + c)$, etc.
 17. 19. 18. 9. 19. 9.
 20. 72; remove brackets and combine numerical quantities.
 21. $4\frac{1}{3}$. 22. 4.
 23. 3; equation is $\frac{7}{18}x + \frac{7}{18}x + \frac{3}{7}x - \frac{11}{14}x = \frac{1}{18} + \frac{2}{7} + \frac{3}{11} + \frac{1}{6}$; or
 $\frac{211}{14}x - \frac{11}{14}x = 8051 \div 66 \times 91$;
i. e. $8051x \div 77 \times 234 = 8051 \div 66 \times 91$, etc.
 24. $(2ab^2 - 5a) \div (2a - 2b + 3)$. 25. 8.

EXERCISE XXXI. (PAGE 72.)

1. 240. 2. 12 miles. 3. $8\frac{1}{2}$ miles. 4. 8 men.
 5. $na^2 + nb \div a$. 6. $mn(a - b) \div (mn - m - n)$.
 7. Price = $\$(22x - 21z)y \div 20x(x - z)$. 8. 50 gal.
 9. One-third. 10. 188 oz.; $\frac{1}{20}(x + 32) = \frac{1}{12}(x - 56)$, where
 $x = \text{wt. of lump}$.

11. 30 eggs. 12. 4. 13. 40. 14. \$78 $\frac{1}{2}$.
 15. 16200, 23000. 16. 63. 17. 30 gal. 18. $1080 \div 251$ miles.
 19. B in $ac \div (a - b)$ days, A in $ac \div (c - a + b)$.
 20. 22 gals., 9s. 21. \$3. 22. 37, 38, 39. 23. 6, 9, 18. 24. 235.
 25. Let x = increase of rate, then $c \div a + x = c \div a - b$,
 $x = a^2b \div (c - ab)$.
 26. 7, 8, 9. 27. $2pqr \div (pq + qr + rp)$.
 28. $(ma - b) \div (m - 1)$, $m(b - a) \div (m - 1)$.
 29. 432. 30. $n(m - p) \div p$.
 31. $20\frac{2}{3}$, $24\frac{2}{3}$, $11\frac{1}{3}$, $44\frac{2}{3}$; if $x = 1$ st part, $x + 4 = 2$ d, $\frac{1}{2}(x + 2) = 3$ d,
 $2(x + 2) = 4$ th, and their sum is 100.

EXERCISE XXXII [b]. (PAGE 78.)

1. $169a^2 - 52a + 4a^2$; $225x^2 - 15ax + \frac{1}{4}a^2$;
 $441x^2y^2 + 126x^2y + 9x^2$; $144a^2b^4 - 144a^3b^3c + 36a^4b^2c^2$.
 2. $\frac{4}{3}x^4 + x^2y^2 + \frac{9}{16}y^4$; $18\frac{1}{16}a^4b^2 - 2a^3b^3 + \frac{1}{8}\frac{6}{9}a^2b^4$;
 $289x^4y^6z^8 - 2x^6y^6z^8 + \frac{1}{8}\frac{1}{9}x^6y^6z^4$.
 3. 1,024,144; 1,096,004; 12321; 5625; 2401.
 4. $25a^{116}b^{14} + 60a^{149}b^{12} + 36a^{182}b^{10}$; $a^{224} - 2a^{112}b^{20} + b^{40}$;
 $5929a^{154} + 13552a^{77}b^{88} + 7744b^{176}$.
 5. $169x^2 - 4$; $\frac{1}{4}x^2 - \frac{1}{400}$; $4x^4 - \frac{1}{4}\frac{1}{4}y^4$.
 6. $49x^{14} - 256x^2y^2$; $\frac{1}{4}x^{14} - \frac{1}{8}x^2y^2$; $x^{154} - y^{176}$.
 7. 999,856; 9879; 4875; 2499.
 8. $x^{224} - b^{400}$; $25a^{116}b^{144} - 36a^{182}b^{114}$; $5929a^{154} - 7744b^{176}$.
 9. $4x^2$. 10. $a^2 + b^2 + 2ab - c^2$; $x^2 - 2xy + y^2 - z^2$.
 11. $4a^2 - b^2 + 6bc - 9c^2$; $y^2 - 4x^2 + 12xz - 9z^2$.
 12. $(w + y)^2 - (x + z)^2$; $(s + t)^2 - (u + r)^2$.
 13. $(a + d)^2 - (2b - 3c)^2$; $(3y + z)^2 - (x - 2k)^2$.
 14. $2m^2 + 6ms - 8p^2 + 4pk - 12ps - 2mk$.

EXERCISE XXXIII [a]. (PAGE 80.)

6. $1 + 2x + 3x^2 + 2x^3 + x^4$; $1 - 2x + 3x^2 - 2x^3 + x^4$;
 $1 + 4x + 6x^2 + 4x^3 + x^4$; $1 - 4x + 6x^2 - 4x^3 + x^4$.
 7. $16 + x^2 + 4y^2 + 8x - 16y - 4xy$; $25 + y^2 + 9z^2 - 10y - 30z + 6yz$;
 $1 - 2x - x^2 + 2x^3 + x^4$; $x^4 + y^4 + z^4 + 2x^2y^2 + 2y^2z^2 + 2z^2x^2$.

14. \$78\frac{5}{8}\$.

1080 ÷ 251 miles.

3, 9, 18. 24. 235.

$= c \div a - b,$

$d, \frac{1}{2}(x+2) = 3d,$

E 78.)

$+ 36a^4b^2c^2.$

;

$+ b^{40};$

$744b^{176}.$

$- z^2.$

80.)

$x^4;$

$+ x^4.$

$0y - 30z + 6yz;$

$+ 2y^2z^2 + 2z^2x^2.$

8. $1 + 2x^2 + 6x^3 + x^4 + 6x^5 + 9x^6;$
 $1 - 2x^2 + 6x^3 + x^4 - 6x^5 + 9x^6;$
 $4 - 4y + 9y^2 - 4y^3 + 4y^4; 4x^4 + y^2 + 1 + 4x^2y - 2y - 4x^2.$
9. $1 - 2x + 5x^2 - 4x^3 + 4x^4; 1 + 2x - 5x^2 - 6x^3 + 9x^4;$
 $4a^4 - 7a^2 + 4 - 4a^3 + 4a; 1 + 2a^2 + 2a^3 + 2a^5 + a^6.$
10. $1 + x^2 + b^2y^2 + 2x + 2by + 2bxy;$
 $1 + a^2x^2 + b^2y^2 + 2ax + 2by + 2abxy;$
 $1 + a^2x^2 + b^2y^2 - 2ax - 2by + 2abxy;$
 $1 - 2ax^2 + 2bx^3 + a^2x^4 - 2abx^5 + b^2x^6.$
11. $1 + 2x + 3x^2 + 4x^3 + 3x^4 + 2x^5 + x^6;$
 $1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6;$
 $1 - 2x - x^2 + 3x^4 + 2x^5 + x^6.$
12. $1 - 4ax + 10a^2x^2 - 12a^3x^3 + 9a^4x^4;$
 $x^6 - 6x^5 + 13x^4 - 14x^3 + 10x^2 - 4x + 1;$
 $x^6 - 4x^5 + 10x^4 - 4x^3 - 7x^2 + 24x + 16.$
13. $4a^2 + b^2 + 4c^2 - 4ab + 8ac - 4bc;$
 $a^2 + \frac{1}{4}b^2 + \frac{1}{4}c^2 - ab + ac - \frac{1}{2}bc;$
 $\frac{1}{4}a^2 + \frac{1}{4}b^2 + c^2 - \frac{1}{2}ab - bc + ac;$
 $\frac{1}{4}a^2 + b^2 + \frac{1}{2}c^2 - ab - \frac{2}{3}bc + \frac{1}{3}ac.$

[b.]

1. $(2x+y)^2 \div 160.$ 2. $(a+b-c)^2 \div 100.$ 3. $(x+2y+3w+4z)^2.$
 4. $6(ab+bc-ca)^2.$ 6. $4(w^2+x^2+y^2+z^2).$ 7. $8x^3y.$
 8. $a^5 + 2a^4 + 3a^4 + 2a^2 + 1.$
 10. $2a^2b^2 + 2b^2c^2 + 2c^2a^2 - a^4 - b^4 - c^4;$ see Ex. 3, p. 128.

EXERCISE XXXIV [b]. (PAGE 84.)

4. $(3x+4y)^2 - 25z^2; (2a+4c)^2 - 9b^2.$
5. $\{(x^3+2x^2+4)-3x\} \times \{(x^3+2x^2+4)-5x\}$
 $= x^6 + 4x^5 - 4x^4 - 8x^3 + 31x^2 - 32x + 16;$
 $\{(x+z+w)+y\} \{(x+z+w)+3y\}$
 $= (x+z+w)^2 + 4y(x+z+w) + 3y^2.$
6. $x^3 + 9x^2 + 26x + 24; x^3 + 14x^2 + 55x + 42;$
 $x^3 + 9x^2 + 23x + 15.$
7. $x^3 - 9x^2 + 26x - 24; x^3 - 14x^2 + 55x - 42;$
 $x^3 - 9x^2 + 23x - 15.$
8. $x^3 + 3x^2 - 10x - 24; x^3 - 12x^2 + 29x + 42;$
 $x^3 + x^2 - 17x + 15.$

9. $8x^3 + 12x^2 + 22x + 6$; $8x^3 - 12x^2 + 22x - 6$;
 $8x^3 - 4x^2 - 10x + 6$.
10. $x^4 + x^3(y + z + w + k) + x^2(wy + wz + wk + yz + yk + zk)$
 $+ x(yzw + yzk + zwk + ykw) + yzkw$;
 $x^4 - (a + b + c + d)x^3 + (ab + ac + ad + bc + bd + cd)x^2$
 $- (abc + abd + acd + bcd)x + abcd$.
11. $w^3 + 3w^2r + 3wr^2 + r^3$; $w^4 + 4w^3r + 6w^2r^2 + 4wr^3 + r^4$;
 $8w^3 + 12w^2r + 6wr^2 + r^3$; $w^4 + 8w^3r + 24w^2r^2 + 32wr^3 + 16r^4$;
 $w^3 - 3w^2r + 3wr^2 - r^3$; $w^4 - 4w^3r + 6w^2r^2 - 4wr^3 + r^4$.
12. $k^5 + 15k^4s + 90k^3s^2 + 270k^2s^3 + 405ks^4 + 243s^5$;
 $a^6 - 12a^5b + 60a^4b^2 - 160a^3b^3 + 240a^2b^4 - 192ab^5 + 64b^6$;
 $8a^3 - 6a^2w + \frac{3}{2}aw^2 - \frac{1}{8}w^3$; $\frac{1}{8}a^3 + \frac{3}{2}a^2w + 6aw^2 + 8w^3$;
 $27a^3 - 9a^4 + a^2 - \frac{1}{8}$.
13. $1320a^3b^3$; $-22680a^4b^3$; $-2a^2x^3$.
14. $1485a^2b^{53} + 55ab^{54} + b^{55}$; $2145x^2y^{64} + 66xy^{65} + y^{66}$;
 $-6655a^2 + 121a - 1$.
15. $54a^2b^2$; $540a^3b^3$; $1680a^4$. 16. 1.21662924; 1.7101875.

EXERCISE XXXV [a]. (PAGE 87.)

1. $x^4 + 2x^3 - 85x^2 - 86x + 1680$.
2. Write k for $x + a$, m for $x + b$, \therefore product $= k^4 + k^2m^2 + m^4$
 $= 3x^4 + 6x^3(a+b) + x^2(7a^2 + 4ab + 7b^2)$
 $+ x(4a^3 + 2a^2b + 2ab^2 + 4b^3) + (a^4 + a^2b^2 + b^4)$.
3. $a^2b^3 + c^2d^3 - a^2c^3 - b^2d^3$. 4. $a^3 + b^3 + c^3 - 3abc$.
5. $x^5 - px^4 + qx^3 - qx^2 + px - 1$.
6. $a^3(x^3 - 1) - a^2(x^3 + x^2 - 2) + a(4x^2 + 3x + 2) - 3(x + 1)$.
7. $w^2 - z^2$. 8. $8x^3$. 9. $24xyz$. 10. $zw + xy$.
11. $6xyz$. 12. $4x^2y^2$. 13. See p. 85, H, (3).

EXERCISE XXXVI [b]. (PAGE 88.)

7. $a + b - c$; $x - 2y - 3z$. 8. $a^2 - 2ab + b^2$; $a^2 + ab + b^2$.
9. $1 + 2x + 3x^2$; $3a^2 + 2a + 3$. 12. $x + 4$; $2x - 3b$.
13. $a + 8b$; $2a - 7b$. 14. $1 + a^2$. 15. $x^2 - 2x + 1$.

[c.]

1. $\frac{3a}{5} - \frac{5}{3a}$; $\frac{a}{2b} - 2$; $\frac{8m}{3w} + 2$.

3. $\frac{1}{2}x^2 + \frac{1}{2}x - 1$; $x^2 + x - \frac{1}{2}$; $x^2 + 2x + 1$.
 3. $2x + 3y - 5z$; $2x^2 - x + 1$. 4. $\frac{4y}{x} - 4 + \frac{x}{y}$; $x - 2 - \frac{1}{x}$.
 5. $\frac{3a}{b} - \frac{1}{5} + \frac{2b}{3a}$. 6. $4a - 3b$; $\frac{2}{x^2} - 3x$.
 7. $1 - 2x + 3x^2$; $\frac{1}{2}x - 1$. 8. $a + 2b - c$. 9. $\frac{x}{3} - 1 + \frac{3}{x}$; $\frac{x}{3} + 2$.
 10. Cube both sides by formula G (2), p. 85.

EXERCISE XXXVII.

1. $a^2b + b^2a$; $a(a+b)^2 + b(b+a)^2$;
 $ab(b-c) + bc(c-a) + ca(a-b)$; $a^2bc + b^2ca + c^2ab$;
 $a(b+c) + b(c+a) + c(a+b)$.
 2. $(a-b)(b-c) + (b-c)(c-a) + (c-a)(a-b)$;
 $a^2(b-c) + b^2(c-a) + c^2(a-b)$;
 $a(b-c)^2 + b(c-a)^2 + c(a-b)^2$;
 $(x-a)(b-c)^2 + (x-b)(c-a)^2 + (x-c)(a-b)^2$.
 3. $a^3 + b^3 + c^3 + d^3$;
 $a^2(bc + bd + cd) + b^2(ac + ad + cd)$
 $+ c^2(ab + ad + bc) + d^2(ab + bc + ac)$;
 $a^2(b+c+d) + b^2(c+d+a) + c^2(d+a+b) + d^2(a+b+c)$;
 $a+b+a+c+a+d+b+c+b+d+c+d$;
 $ab+ac+ad+bc+bd+cd$;
 $a^2(a-b) + b^2(b-c) + c^2(c-d) + d^2(d-a)$;
 $(a-b)^3 + (a-c)^3 + (a-d)^3 + (b-c)^3 + (b-d)^3 + (c-d)^3$.
 4. $(a-b)^2(b-c)^2 + (a-c)^2(c-d)^2 + (a-b)^2(b-d)^2 + (b-c)^2(c-d)^2$;
 $(x-a)(b-c)^2 + (x-b)(c-d)^2 + (x-c)(d-a)^2 + (x-d)(a-b)^2$.
 13. $a, b, -c$; $a, -b, c$; $a, -b, -c$. 14. a, b, c ; a, b .
 15. a, b, c ; a, b ; $a, -b$. 16. a, b, c ; ax and by , x and y .
 17. a and b ; a, b, c . 18. a, b, c . 19. a, b ; a, b .
 20. a, b, c . 21. a^2b . 22. a^4, a^3b, a^2b^2, abc^2 .
 23. x^3, x^2y, xyz . 24. a^3b . 25. ab^3, ab^2c .
 26. xy^2, xyz . 27. x^3, x^2y ; x^4, x^3y, x^2y^2 ; x^5, x^4y, y^3y^2 .
 28. x^3, x^2y, xyz . 29. a^4, a^3b, a^2b^2, a^2bc ; $x^5, x^4y, x^3yz, x^2y^2z, x^3z^2$.
 30. a^3, a^2b, abc ; $x^7, x^6y, x^5y^2, x^4y^3$.
 31. $a^3 + b^3 + c^3 + d^3 - 3(abc + abd + bcd + cda)$.

32. $a + b - c$; $a - b + c$; $-a + b + c$; $a - b - c$.

33. $\frac{1}{2} \{ (a - b)^2 + (b + c)^2 + (c + a)^2 \}$;

$\frac{1}{2} \{ (a + b)^2 + (b + c)^2 + (c - a)^2 \}$;

$\frac{1}{2} \{ (a + b)^2 + (b - c)^2 + (c + a)^2 \}$;

$\frac{1}{2} \{ (a + b)^2 + (b - c)^2 + (c + a)^2 \}$;

the three expressions are derived from the *first* by, respectively, substituting $-c$ for c , $-b$ for b , $-a$ for a ; observe, also, that $(-a - c)^2 = + (a + c)^2$.

EXERCISE XXXVIII. (PAGE 96.)

1. $3(a^2 + b^2 + c^2) - 2(ab + bc + ca)$. 2. 0. 3. $2(xy + yz + zx)$.
4. $6(a^2 + b^2 + c^2) - 2(ab + bc + ca)$.
5. $2(x^2 + y^2 + z^2 - yx - yz - zx)$.
6. $14(a^2 + b^2 + c^2) - 14(ab + bc + ca)$.
7. $4(a^2 + b^2 + c^2 + d^2)$. 8. $4(a^2x^2 + b^2y^2 + c^2z^2)$.
9. $2(a^3 + b^3 + c^3) + 6(a^2b + \text{etc.}) - 12abc$.
10. $a^2 + b^2 + c^2 + d^2$. 11. $3(a^2 + b^2 + c^2 + d^2) + 2(ab + \text{etc.})$.
12. 0. 13. $6abc$. 14. $abc(a + b + c)$.
15. $4(x^4 + y^4 + z^4) + 24(a^2b^2 + b^2c^2 + c^2a^2)$.
16. NOTE.—The first term in each of the binomial *factors* should have *index* 2; i. e., a^2 for a , etc. Multiply out, or use identity,
 $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.
17. Multiply out and subs. for s .
18. $rs = (a + b)^2 - (c - d)^2$, the other pairs by symmetry; result is $4(ab + ac + ad + bc + bd + cd)$.
20. Type terms are a^4 , $2a^3(b + c)$, a^2b^2 , and both expressions reduce to same form. Or, use identity, Ex. 7, p. 105, putting $a - b$ for a , $b - c$ for b , and $\therefore a - c$ for $a + b$.

EXERCISE XL. (PAGE 98.)

1. $(a - b)(x + 2y)$. 2. $(a + b)(2x - 3y)$.
3. $(a + x)(a - b)$. 4. $(c - d)(ab - c)$.
5. $(m + n)(x^2 - a)$. 6. $(a + b)(a - c)$.
7. $(a + b)(3x + y)$. 8. $(a - bc)(1 - x)$.
9. $(a - b)(c + y)$. 10. $(a + x)(a + b)$.
11. $(3a - b)(x - y)$. 12. $(7 - x)(a - bc)$.

13. $(r-s)(3p+q)$.
 15. $(3x-a)(2x+y)$.
 17. $(x-1)(3bx-1)$.
 19. $(a-1)(a^2+1)$.
 21. $(x^2+a^2)(a-3c)$.
 23. $(x^2-1)(2a^2-1)$.
 25. $(a^2x^2-c)(a^2x^2-b)$.
 27. $(x^2-a^2)(x^2+ax+a^2)$.
 29. $(a+b)(ax+by+c)$.
 31. $(a-1)(a+b)$.
 33. $(1-b)(a-b+c)$.
 35. $(1-x^2)(1+x^2+p+q)$.
 37. $(a+b-c)(d-e+f)$.
14. $(1-a)(1-b)$.
 16. $(a^2-1)(a+1)$.
 18. $(xy-z)(a+bc)$.
 20. $(x+f)(2a+b)$.
 22. $(x-y)(x-3)$.
 24. $(b-1)(c-1)$.
 26. $(3b^2-1)(1-3a^2)$.
 28. $(a-b)(x-y+z)$.
 30. $(ax^n-b)(bx^n+a)$.
 32. $(2+x^n)(3-y^n)$.
 34. $(a-x)(2pq-3bf)$.
 36. $(2p^n-3q^n)(r^n-2s^n)$.
 38. $(1+p+q)(1-a+b)$.

EXERCISE XLI. (PAGE 100.)

11. $\left(\frac{a}{b}-\frac{b}{a}\right)^2$; $(1-x^m)^2$; $\{(2x-3y)-(2x+3y)\}^2$.
 12. $\{(a^2+ab+b^2)-(a^2-ab+b^2)\}^2$; $\left\{\left(\frac{x}{y}\right)^m-\left(\frac{y}{x}\right)^m\right\}^2$.
 13. $(x+y+z)^2$; $(p-q+r)^2$.
 14. $(a-2b+3c)^2$; $(1-x+y)^2$.
 15. $(3a+2b+c)^2$; $(2a^2-3a+4)^2$.
 16. $(3ax+2by+cz)^2$.
 17. $(2a^2-3b+4c)^2$; $(a^2-b^2-c^2)^2$.
 18. $\pm 4xy$; $\pm xy$; x^2y^2 ; $-10xy$; $\pm 4x^2y^2$.
 19. $\pm 6ay$; $\pm 10a^2b$; $\pm 12x^2y^2$; $\pm 2a^2b^2$; a^2 .
 20. a^4 ; z^2 ; $\frac{1}{4}$; $\frac{1}{4}$; 4 ; b^2 .
 21. $\frac{1}{4}b^2$; $\pm 4bxy^2$; ± 2 ; ± 2 ; x^4 .
 22. 7 ; $\frac{1}{4}$; $b^2 \div 4a^2$; $25 \div 4$; $49 \div 4$.
 23. $81 \div 16$; $x^2 \div 4$; $x^2 + 13$; $-c + \frac{1}{4}b^2$.

EXERCISE XLII [b]. (PAGE 103.)

NOTE.—The two factors in each case are expressed with the double sign \pm .

1. $a+b \pm c$; $2(x+y) \pm z$; $a \pm (y+z)$; $2 \pm (a+b)$.
 2. $p+2q \pm r$; $4x \pm (a+3b)$; $2m \pm (p-q)$; $2x(-4y)$.
 3. $1 \pm (b-c)$; $a+b+c \pm x$; $(8+x)(10-x)$; $b-c \pm (a-x)$.

4. $3\{2(a^2 - bc) \pm (b^2 - ac)\}; a - 5b \pm 1; 1 \pm (x - y + z);$
 $(a^4 + b^4)(a^2 + b^2)(a + b)(a - b); (a - 3c)(a + 4b + 3c).$
5. $(-a + b - 4c)(3a - 5b + 4c);$
 $(1 - a + b)(1 - a - b)(1 + 2a - a^2 + b^2);$
 $(12x - 1)(2x + 7).$
6. $(x - z \pm y)(x + z \pm y); 4(x + z)(y + u);$
 $\{x \pm (y + z)\}\{x \pm (y - z)\}.$
7. $(x - z) \pm (y - u); a \pm (x - y); x \pm (y + z).$
8. $x \pm (y - z); x \pm (y + z); x + z \pm y; x^2 \pm (x - 1).$
9. $(x + a) \pm (y + z); (a - c) \pm (b - d);$
 $(a^3 + b^3)(a^2 + b^2)(a + b)(a - b);$
 $(a^2 + 6a + 5)(a^2 + 2a + 3).$
10. $a - b \pm (x + y); a^2 + a \pm (b^2 - b).$
11. $(x + b)(a \pm x); \{a - d \pm (b - c)\}; ab \pm c(a - b).$
12. $\{c \pm (a - b)\}\{a + b \pm c\}; x^2 + y^2 \pm (z^2 + 1); a - d \pm (b - c).$
13. $2a \pm (b - 3c); b \pm (2a - 3c); 2a \pm (b + 3c).$
14. $3c \pm (2a - b); (a + c) \pm (b + d); (a + d) \pm (b + c).$
15. $(b + c) \pm (a + d); (a + d) \pm (2b - 3c).$
16. $3c + d \pm (a - 2b); (a - 3c) \pm (2b - d).$
17. $\{a + d \pm (b - c)\}\{b + c \pm (a - d)\}.$
18. $(x^2 + 1 \div y^2)(x + 1 \div y)(x - 1 \div y); x^4 \pm \frac{1}{16}, \text{ etc.};$
 $x^3(x^4 - 25) - \frac{1}{4}(x^4 - 25) = (x^4 - 25)(x^3 - \frac{1}{4}), \text{ etc.};$
 $(x^4 - 16)(x^3 + 1), \text{ etc.}$

EXERCISE XLIII. (PAGE 105.)

- | | |
|-------------------------------|---|
| 1. $3x^2 + y^2 \pm xy.$ | 2. $4a^2 - b^2 \pm 3ab.$ |
| 3. $3a^2 + b^2 \pm 5ab.$ | 4. $5m^2 + 4n^2 \pm 7mn.$ |
| 5. $x^2 + 1 \pm x.$ | 6. $x^2 + 4 \pm 2x.$ |
| 7. $x^3 + 25 \pm 5x.$ | 8. $x^2 + \frac{1}{4} \pm \frac{3}{4}x.$ |
| 9. $x^2 - y^2 \pm 3xy.$ | 10. $x^2 + \frac{1}{9} \pm \frac{2}{3}x.$ |
| 11. $a^2 - y^2 \pm 2ay.$ | 12. $m^2 - n^2 \pm 4mn.$ |
| 13. $9a^2 + b^2 \pm 3ab.$ | 14. $4a^2 + b^2 \pm 6ab.$ |
| 15. $5p^2 - 4q^2 \pm pq.$ | 16. $9x^2 - y^2 \pm 4xy.$ |
| 17. $2x^2 - 1 \pm 2x.$ | 18. $\frac{1}{2}x^2 + y^2 \pm xy.$ |
| 19. $x^2 + 2a^2y^2 \pm 2axy.$ | 20. $2a^2 \pm y^2 \pm \frac{7}{2}ay.$ |

$$(x - y + z);$$

$$(a + 4b + 3c).$$

21. $x^4 + y^4 \pm x^2y^2$, etc.; $x^4 + 1 \pm x^2$.
 22. $a^2x^4 + 1 \pm ax^2$; $x^4 + 2y^4 \pm 2xy$.
 23. $(a + b)^2 + c^2 \pm 3c(a + b)$; $1 + 2x^2 \pm 2x$.
 24. $4x^2 + 2(y - z)^2 \pm 5x(y - z)$; $1 + 5z^4 \pm 3z^2$.
 25. $1 + 2a^4 \pm 2a^2$; $a^2 + 9b^2 \pm 9ab$.
 26. $2(1 + a + a^2)^2$; $x^2 + 1 \div y^2 \pm x + y$.
 27. $x^3 + 1 \div 2y^2 \pm x + y$; $a^2 + 2 \div a^2 \pm 2$;
 $(a + b)^2 + (a - b)^2 \pm (a^2 - b^2)$.
 28. $c^2 + 2(a + b)^2 \pm 2c(a + b)$; $1 \div a^2 + 1 \div b^2 \pm 1 \div ab$;
 $3 \div a^2 + 1 \div b^2 \pm 3 \div ab$.

EXERCISE XLIV [a]. (PAGE 108.)

17. $(m^3 + 21)(m^3 + 19)$.
 18. $(a^2x + 39)(a^2x + 1)$.
 19. $(x^n + 7)(x^n + 12)$.
 20. $(x + 17)(x + 23)$.
 21. $(x^n + 12)(x^n + 4)$.
 22. $(x + 33)(x + 27)$.
 23. $(a^2 + 27)(a^2 + 13)$.
 24. $(a + 18b)^2$.
 25. $(a^2x + 81)^2$.
 26. $(x - 4)^2$.
 27. $(x - 15)^2$.
 28. $(x - 19)^2$.
 29. $(x - 20)^2$.
 30. $(x - 50)^2$.
 31. $(x^3 - 5)(x^3 - 25)$.
 32. $(m - 17n)(m - 5n)$.
 33. $(x - 13y)^2$.
 34. $(x^2 - 5y^2)(x^2 - 4y^2)$.
 35. $(a - 27b)(a - 2b)$.
 36. $(4 - x)(3 - x)$.
 37. $(26 - ab)(5 - ab)$.
 38. $(a - 25)(a - 15)$.
 39. $x^2 + 1 \pm \frac{1}{3}x$.
 40. $(x^3 - 27)(x^3 - 8)$.
 41. $3x(x - 2)(x - 8)$.
 42. $a(x - 5)(x - 6)$.
 43. $x^2 \pm 60 \pm 17x$.
 44. $(x^n - 7)(x^n - 37)$.
 45. $(a + b - 4)(a + b - 3)$.
 46. $(13 - ax)(11 - ax)$.
 47. $(1 - 8x^2y^2)(1 - 51x^2y^2)$.
 48. $(a - 27b)^2$.
 49. $x^2(a - 15bx)(a - 5bx)$.
 50. $(m - 19)^2$.
 51. $(p - 27q)^2$.
 52. $\{(x - y)^n - 33\} \{(x - y)^n - 11\}$.

[b.] (PAGE 109.)

1. $(a^2 + 1)(a^2 - 2)$.
 2. $(a + 3)(a - 2)$.
 3. $(x - 3)(x + 2)$.
 4. $(x - 16)(x + 3)$.
 5. $(x + 12)(x - 7)$.
 6. $(y + 12)(y - 5)$.

7. $(a + 20)(a - 7)$.
9. $(x + 12)(x - 11)$.
11. $(y^3 - 10a^2)(y^2 + 5a^2)$.
13. $3(az^2 - 14)(az^2 + 1)$.
15. $(abc + 11)(abc - 2)$.
17. $(x^2 - 48)(x^2 + 8)$.
19. $(x + y - 19)(x + y + 18)$.
21. $(x^{2n} + 4)(x^{2n} - 3)$.
23. $(13 - ab)(5 + ab)$.
25. $3y(a + 14bx)(a - 2bx)$.
27. $(3x + 7)(3x + 5)$.
29. $(7a - 8b)^2$.
31. $x^4(8y^3 - 10z)^2$.
32. $(a^2 - 40b^2)(a^2 + 5b^2)$.
33. $(11x^2 - 13y)^2$.
34. $3(x^2 + y^2)(3x^2 - 4y^2)$; where $x = a - b$ and $y = c$.
35. $(8x^n - 2b^n)(8x^n + b^n)$.
37. $(\frac{3}{4}a + 7b)(\frac{3}{4}a - 8b)$.
39. $(\frac{4}{x} + 19)(\frac{4}{x} - 20)$.
8. $(a + 25b)(a - 12b)$.
10. $(x - 10)(x + 2)$.
12. $(ab - 4)(ab + 1)$.
14. $(a^4 - 20)(a^4 + 5)$.
16. $(a^2b^3 - 30)(a^2b^2 + 3)$.
18. $(x^n - 16)(x^n + 3)$.
20. $\{a - 30(b + c)\} \{a + 12(b + c)\}$.
22. $(20 + a)(19 - a)$.
24. $(12 - m)(17 + m)$.
26. $(2x + 7)(2x + 5)$.
28. $(2x^2y - 7z^2)(2x^2y + 6z^2)$.
30. $x(2b - y)(2b - 5y)$.
36. $(\frac{1}{2}x^2 + 7)(\frac{1}{2}x^2 - 6)$.
38. $(\frac{3}{x} - 7)(\frac{3}{x} + 9)$.
40. $(5x^2 + 21)(5x^2 - 31)$.

EXERCISE XLV. (PAGE 112.)

1. $(2x + 1)(2x + 3)$.
3. $(3x + 3)(5x + 4)$.
5. $(2x + 5)(3x + 4)$.
7. $(4a + 9)(a + 1)$.
9. $(x + 5)(4x + 3)$.
11. $(4x - 3)(3x + 2)$.
13. $(4x + 7)(3x - 5)$.
15. $(3x + 2)(2x - 1)$.
17. $(3x + 4)(5x - 2)$.
19. $(5x + 2y)(3x - 5y)$.
21. $(3m + 20)(2m - 19)$.
23. $(3x + 7y)(4x - 5y)$.
25. $(5x^2 - 1)(4x^2 + 1)$.
27. $(12x - 7)(2x + 3)$.
29. $(8a + b)(3a - 4b)$.
2. $(4x + 1)(x + 3)$.
4. $(3x + 2)(2x + 1)$.
6. $(2x + 7y)(4x + 3y)$.
8. $(1 + m)(7 + 3m)$.
10. $(x + 7)(3x + 2)$.
12. $(4x + 3)(3x - 2)$.
14. $(4x - 7)(3x + 5)$.
16. $(5x - 1)(2x - 3)$.
18. $(x - 7)(7x - 1)$.
20. $(a^2 - 19)(a^2 + 17)$.
22. $(2a + 20)(3a - 19)$.
24. $(3 - 12x)(5 + 11x)$.
26. $(15a - 1)(a + 15)$.
28. $(6 - y)(3 - 5y)$.
30. $(8 - 9y)(3 + 8y)$.

$(a - 12b)$.
 $x + 2)$.
 $ab + 1)$.
 $(a^4 + 5)$.
 $0) (a^2b^2 + 3)$.
 $(x^8 + 3)$.
 $+ c) \} \{ a + 12(b + c) \}$.
 $19 - a)$.
 $(17 + m)$.
 $2x + 5)$.
 $z^2) (2x^2y + 6z^2)$.
 $(2b - 5y)$.
 33. $(11x^2 - 13y)^2$.
 and $y = c$.
 $(\frac{1}{2}x^2 - 6)$.
 $(\frac{3}{2} + 9)$.
 $(5x^2 - 31)$.

112.)
 $x + 3)$.
 $2x + 1)$.
 $(4x + 3y)$.
 $+ 3m)$.
 $+ 2)$.
 $x - 2)$.
 $x + 5)$.
 $x - 3)$.
 $- 1)$.
 $^2 + 17)$.
 $(a - 19)$.
 $+ 11x)$.
 $+ 15)$.
 $5y)$.
 $+ 8y)$.

- | | |
|------------------------------------|------------------------------------|
| 31. $(28x^2 - 25)(x^2 + 5)$. | 32. $4(14x + 5y)(x - y)$. |
| 33. $4(7x - 5y)(2x - y)$. | 34. $4(14x - 5y)(x + y)$. |
| 35. $(8a - 5b)(7a - 4b)$. | 35. $2(28y + 1)(y - 10)$. |
| 37. $(8y + 5z)(9y - 8z)$. | 38. $(9y + 3a)(4y - 5a)$. |
| 39. $(56a^2 + 4b^2)(a^2 - 5b^2)$. | 40. $(56a - 5b)(a - 4b)$. |
| 41. $(13x + 12y)(3x - 4y)$. | 42. $(3x + y)(13x - 11y)$. |
| 43. $(39x - 26)(x + 1)$. | 44. $(12x + 13y)(5x - 8y)$. |
| 45. $(1 - 13x^2)(1 + 11x^2)$. | 46. $(a^n - 13b^n)(a^n + 11b^n)$. |
| 47. $(3x^3 - 21)(4x^3 + 11)$. | 48. $(17x - 1)(x + 17)$. |

EXERCISE XLVI. (PAGE 114.)

- | | |
|--|-------------------------------------|
| 1. $(x - a)(x^2 - 2x - 1)$. | 2. $(x - a)(x^2 - px + q)$. |
| 3. $(my - n)(ay^2 + by - c)$. | 4. $(2b - c)(x^2 - 2bx + b)$. |
| 5. $(nx - a)(x^2 - x - 1)$. | |
| 6. $(bx - a)\{(m + 1)b^2x^2 + (m + 1)(n + 1)abx + (n + 1)a^2\}$;
multiply out, take m -terms for one group, etc. | |
| 7. $(y - b)(y - a)^2$. | 8. $(x - b)(x - a)(x + 2b)$. |
| 9. $(x + p + q)(x + q - p)(x - 2q)$. | 10. $(x - a)(x + b)(x + 3)$. |
| 11. $(x + b)\{x(x - 1) - a(x + 1)\}$. | 12. $(2x - a)(2x^2 + 4x - 3)$. |
| 13. $(py + q)(y^2 - y + 1)$. | 14. $(mx - n)(px^2 + qx - r)$. |
| 15. $(mx - n)(ax^2 - cx - b)$. | 16. $(px - q)(3x^2 - cx - b)$. |
| 17. $(x^2 - px + q)(ax^2 + bx - c)$. | 18. $(2x + 3c)(x^2 + ax - 2b)$. |
| 19. $(2x + 3c)(x^2 - 2ax + 3b)$. | 20. $(ap - bq)(2p^2 + 3pq + q^2)$. |
| 21. $(ap - bq)(3p^2 - pq - 2q^2)$. | 22. $(ax + b)(cx^2 + dx + c)$. |
| 23. $(ax + b)(2cx^2 - dx - 3c)$. | 24. $(3y - ab)(3y - bc)(3y + 5)$. |

EXERCISE XLVII. (PAGE 118.)

- | | |
|-------------------------------------|--------------------------------------|
| 1. $(6x - y + 1)(x - 6y - 1)$. | 2. $(3x + 2y + 1)(2x - 3y - 1)$. |
| 3. $(4a + 5b + 4)(3a - 4b - 5)$. | 4. $(x - y + 3z)(x + 2y - z)$. |
| 5. $(3x + y + 3)(x - 3y + 9)$. | 6. $(2a - 5b + 6c)(3a + 4b - 8c)$. |
| 7. $(3a - b - 7)(4a - 3b + 8)$. | 8. $(7x - y - 1)(x - y + 3)$. |
| 9. $(a + 3y)(a - 4y - 5)$. | 10. $(2x - 5y - 7z)(2x + 3y + 3z)$. |
| 11. $(3x + y - 4z)(3x - 3y - 2z)$. | 12. $(3x - 2y + 3z)(2x - 3y + 2z)$. |
| 13. $(5x - 3y + 2z)(x - y - z)$. | 14. $(a - 2b + 3c)(14a - b - c)$. |
| 15. $(2a - b - 3c)(4a - 3b - c)$. | 16. $(1 - 3x + 4y)(1 + 7x - 5y)$. |

EXERCISE XLVIII. (PAGE 122.)

7. 8. $-8a^3$. 9. -205 . 10. 1.
 11. $a^3 + pa^2 + qa + r$. 12. -36 . 13. 1555.
 14. $x + 2, x - 3$. 15. $(x + 1)(3x + 2)(2x - 1)$.
 16. Last term should be $52a^3$. 18. $(x - 2)(x - 5)(x + 7)$.
 19. -535 . 20. -800 . 21. 101. 22. 115.
 23. $-a^3 - pa^2 - qa - r$. 24. $abc - 4ab(a + b)$.
 29. -1 . 32. 2. 34. $2(a + b)^3$. 35. 0. 39. 0.
 40. 0. 41. yes; put 1 for $x + y$. 42. 0.
 44. $a^2 + pa + q, a^2 + p'a + q'$.

EXERCISE XLIX. (PAGE 126.)

5. $(p - 1)^2 - (p - 1)(q + 1) + (q + 1)^2; a^n - b^n$.
 6. $x^{10} - x^5a^5 + a^{10}; 1 - (a - b) + (a - b)^2 - (a - b)^3$.
 7. $x^2 - 1 + 1 \div x^2; x^8 - 2x^6 + 3x^4 - 2x^2 + 1$.
 8. $x^2 + y^2 \pm xy; (a^2 + 4b^2)(a + 2b)(a - 2b); 2x(x^2 + 12y^2)$.
 9. $(a + b)(a^2 + b^2 \pm ab);$
 $x^2 - 2y^2$ and $x^8 + 2x^6y^2 + 4x^4y^4 + 8x^2y^6 + 16y^8$.
 10. $(a^2 + bc)(a^4 + 7b^2c^2 - 4a^2bc); (x + 1)^8(x - 1)^8;$
 $(x + 1)(x - 1)(x^2 + 1)(x^4 + 1)$.
 11. $a^3 - (2b^5)^3 = \{a - 2b^5\} \{a^2 + 2ab^5 + 4b^{10}\}; (a - b)ab$.
 12. Expression $= (4a^2 - 9b^2)(x^3 - 8a^3)$, etc.; $\left(a - \frac{1}{2a}\right)\left(a^2 + 1 + \frac{1}{4a^2}\right)$.
 15. Expression $= (x^2 - y^2)(x - y)^2 = 128a^3b^3(a^2 + b^2)$.
 16. $x - 1$, factor dividend.
 17. $(a^4 - a^2x^2 + x^4)(a^8 + a^4x^4 + x^8)$.
 19. Factor and divide by $a + 1 \div a, \therefore a^2 - 1 + 1 \div a^2 = 0,$
 $\therefore a^2 + \frac{1}{a^2} = 1, \therefore a^2 + \frac{1}{a^2} + 2 = 3$, etc.
 20. Expression $= (1 - x)(1 - x^n)$.
 21. Divisor $= (x - 1)(x^2 - x + 1)$ and given expression vanishes
 for each of these factors.
 22. $(x - y)(x^2 + y^2)(x^4 + y^4)$.

EXERCISE L [a]. (PAGE 129.)

1. $3(x+y)(y+z)(z+x)$.
2. $(a-b)(b-c)(a-c)$.
3. $(a-b)(b-c)(a-c)$.
4. $(a-b)(b-c)(c-a)$.
5. $3(x-y)(y-z)(z-x)$.
6. $3(a^2-b^2)(b^2-c^2)(c^2-a^2)$.
7. $(x+y)(y+z)(z+x)$.
8. $-(a-b)(b-c)(c-a)(a+b+c)$.
9. $(a-b)(b-c)(c-a)(a+b+c)$.
10. $(x+y)(y+z)(z+x)$.
11. $3(a^3-b)(b^3-c)(c^3-a)$.
12. $-(a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)$.
13. $(a+b+c)(a^2+b^2+c^2)$.
14. $(x+y+z)^3$.
15. $(a+b+c)^3$.
16. $6abc$. Insert in text $-(a+b)^3$, and read $-$ before $(c+a)^3$.
17. $(a-b)(b-c)(c-a)(ab+bc+ca)$.

[b.] (PAGE 130.)

2. By symmetry; or formula (H) (4), p. 85.
3. By symmetry; or transpose $3abc$, then a is a factor, etc.
4. $5(x+y)(y+z)(z+x)(x^2+y^2+z^2+xy+yz+zx)$.
5. $(a+b-c)(a^2+b^2+c^2-ab+bc+ca)$;
 $(-a+b+c)(a^2+b^2+c^2+ab-bc+ca)$;
 $(-a-b+c)(a^2+b^2+c^2-ab+bc+ca)$.
6. $12\frac{1}{2}$; 2d term should be $13x^3$.
7. Use synthetic division; $4m-12=0$, $m=3$.
8. Given expression $= (x-3)(x+2)(x-5)$, which is true for
all values of x , \therefore coefficient of like powers of x are equal;
i. e., $a=-6$, $b=1$, $c=30$.
9. $b=-6$, $c=5$, $a=12$.
11. $1 \div (a+b+c)$.
12. 2; dividend is 2 (divisor).
13. $16abc(b-c)(a-c)(a-b)$.
15. $-abc(b-c)(c-a)(a-b)$.
16. $-abc(a+b+c)(b-c)(c-a)(a-b)$.

EXERCISE LII [a]. (PAGE 133.)

1. ax ; $x+2$; $2(x-y)^2$.
2. $2(x-a)$; $ab(x-a)(x-b)$.
3. $x+3$; $x+9$.
4. $x-2$; $x+2$.
5. $a+b$; $x+1$; $x-3$.
6. $x+5$; $x+4$; $(x+1)^2$.
7. $x+3$; $x-11y$.
8. $3(x+6y)$; $(x+y)^2$; x^2+4 .

9. $x + 2$; $x + a$. 10. $x - a$; $x - y$.
 11. $x + 3z$; $a + 3$. 12. $a - 1$; $x - a - 4$.
 13. $a + b - c$; $a + b + c$. 14. $a + b + x + y$; $x + a$.
 15. $x - a$; $8(x - 3y)$; $x + a$. 16. $x + a$; $x - 5$; $(1 - x)^2$.
 17. $x^2 + xy + y^2$; $x^2 + a^2$; $x^2 - y^2$.
 18. $3(x - y)(x + y)$; $3(a + b)(a^2 + b^2)$.
 19. $5(p - q)(p + q)$; $x + y$.

[b.]

20. $x + y$. 21. $3x + 1$; $5x - 1$.
 22. $2a + 5$; $a + 5$. 23. $x + 3$; $(x - 1)^2$.
 24. $a^2 + ab + b^2$; $a + b$. 25. $2x + 1$; $x^2 + y$.
 26. $x^2(3x + 2)$. 27. $x^2 + 2y^2 - 2xy$.
 28. $3x + 4a$; unity. 29. 4 ; $1 + \frac{1}{a}$.
 30. $a^2 + \frac{1}{a^2}$. 31. $2x - 1$.
 32. $2a + 3b - c$. 33. $5(x + 2y)$.
 34. $mx + m - x$. 35. $ap - bq$.
 36. $x + 2ab$; omit a in ax^3 . 37. $(a - b)(x + a)$.
 38. $3(2a - 7)$.
 39. $(2ax - y)$; last term of 2nd expression should be $3y^3$.
 40. $(x - 1)^2$.

EXERCISE LIII [a]. (PAGE 141.)

1. $2(x + 1)^2$. 2. $x - 5$. 3. $2x^2 - 3x - 1$.
 4. $7a^2 + 3a - 1$. 5. $y^2 + 8y - 2$. 6. $y^2 - 3y - 5$.
 7. $x^2 - 2x - 3$. 8. $x^2 - 3$. 9. $5x^2 - 1$.
 10. $3x^2 - 2xy + y^2$. 11. $x - 1$. 12. $(x + 2)^2$.
 13. $x(2x^2 + 2xy - y^2)$. 14. $x - 2$.
 15. $(x - 1)(x + 1)$ or $x^2 - 1$.

[b.]

1. $x^2 - 3x + 2$. 2. $x^2 - 13x + 5$.
 3. $a^2 - 8$. 4. $(x - 3)$.
 5. $12x + 5$. 6. $2x^3 - 4x^2 + x - 1$.

7. $x^2 + 3x + 5$. 8. $(x + 1)(x^2 + 1)$.
 9. $a^3 - a^3 - a - 1$. 10. $2y^2 - 7$.
 11. $2x^2 - 3x - 1$. 12. $x^3 + x^3 - 5x + 3$.
 13. $3x - 5a$. 14. $2x^2(2x + 9)$.
 15. $x^2 - 7x - 3$.

EXERCISE LIV [a]. (PAGE 143.)

1. $2 \cdot 2a^3$; $2x^3 \cdot 6yz$; $ab \cdot cx^2y$; $a \cdot 12ab^2$; $4z \cdot 7x^2yz$.
 2. $42a^3b^3$; $4y^2 \cdot x^4$; $y \cdot 10x^4y^3$; $3c \cdot 7a^2b^3c^2$.
 3. $y \cdot x^3y^2$; $2 \cdot 3a^2b^3$; $7a^2 \cdot 3b^2 \cdot 2$; $ab \cdot c$.
 4. $2y \cdot 3x^2y \cdot 2$; $8y^3 \cdot 7x^4y$; $2 \cdot 3x^2y^2 \cdot 2xy$; $ax^3 \cdot 6xy$.
 5. $2 \cdot 3abc \cdot 2$; $3 \cdot 4x^2y \cdot 5y$; $p \cdot 6pq^2 \cdot p$; $ax^m \cdot by^n$.
 6. $1 \cdot a^3b^2c^4$; $xy^3 \cdot 15a^3x^2y$; $n^3 \cdot 3m^3p^4$; p^nq^n .
 7. $3x(a - x)$; $3a^2b(a + b)$; $a(a - b^2)$; $abc(a^2 - c^2)$.
 8. $4a^2x(a + x)$; $21(x + y)(a + b)$; $a(p + q)(p - q)$.
 9. $a(a + b)(b + c)$; $x(x + 1)$; $x^2(x - 3)$; $(x - 1)(x + 1)$; $a^3(a - b)^3$.
 10. $ab(x + a)(x + b)$; $ab(x^2 - a^2)$; $(x - 1)(x + 1)$.
 11. $(x - 2)(x - 1)$; $21(x - 2)(x + 2)$; $x(x + 1)(x + 2)$; $(x + y)^3$.
 12. $(x + 1)(x + 2)(x + 3)(x + 4)$; $(a + b)(a - b)$.
 13. $x(3x - 2)(2x - 5)(x + 7)$; $x^2y(a^2 - b^2)$.
 14. $x^3(x^3 - a^3)$; $x^3(x - a)(x - b)(x - c)$.
 15. $6(x - y)^3$; $6(x + y)^3$; $(x + b)(x^2 - a^2)$.
 16. $(2x - 5)(9x^2 - 1)$; $a^4(a + 3)^2$; $(a - b)^2(a + b)^3$.
 17. $(a + b)^2(a - x)^2$; $(a + b)^2(a^3 + b^3)$.
 18. $(a - x)^2(b - y)^3$; $6(a + 1)^2(a - 1)^2$.
 19. $-(1 - 2x)(1 + 2x)$; $(x^2 - y^2)^2$; $a^nb^n(a - 1)(b + 1)$.
 20. $-6a^nb^n(b^2 - 1)$; $x^2 - y^2$; $(x^2 - 1)(x^2 + 1)$ or $x^4 - 1$.

[b.] (PAGE 144.)

21. $ab(4a^2 - 1)$; $6x(3x - 1)$; $x(x - 2)(x + 2)$.
 22. $(x + 1)(x - 1)^2$; $(x - 1)(x - 2)(x - 4)$;
 $(x - 1)(x + 2)(x - 3)$.
 23. $(x + 3)(x + 4)(x + 5)$; $(x - 1)^2(x + 2)$.
 24. $(a - 1)(a - 2)(a + 2)$; $(x^2 - y^2)^2$ or $(x - y)^2(x + y)^2$.

25. $(x+2)(x-4)(x-10)(x+12)$; $(x+3)(x-3)(x-12)$.
26. $(x-2)(x-4)(x-7)$; $(x+1)(x+3)(x-4)$.
27. $(x+a)(x^2-b^2)$; $(1-x)(1+x)^2$.
28. $(x-a)(x+a)(x-b)$; $(a+b-c)(a+b+c)^2$.
29. $(x-2)^2(x+2)^2$; $(x+3)(x-3)(2x-1)$.
30. $(y+2)(y-3)(3y+1)$; $(2x+3)(2x-3)(3x-2)$.
31. $(2x+3y)(2x-3y)(3x-2y)$; $3(x-1)(3x-1)(3x+2)$.
32. $20a^2y(4a-1)(5a+1)(3a+1)$; $(4a-1)(4a+1)(5a+1)$.
33. $(x+2)(x-2)(3x-7)$; $x^{4n}-y^{2n}$.
34. $(x-2)(x-3)(x-4)(x-5)$; $2ay^2(4x-1)(x+3)(3x-2)$.
35. $(x+y)(x-y)(x^2+y^2)$; $12(x^2-1)^2$.
36. $(x-a)^2(x+a)^2$ or $(x^2-a^2)^2$; $-12x(x-1)(x+1)^2$.
37. $bc(a^2-b^2)$; $x^2(\omega-1)^2(x+1)^2$.
38. $(x+y)(x^2+xy+y^2)(x-y)$ or $(x+y)(x^3-y^3)$; $24(1-x^4)$.
39. $a^4x^4-b^4y^4$; a^8-b^8 .
40. $12a^3(a^2-y^2)$, (x^3+a^3) .
41. $1-x^3$; $(1-2x)^2(1+2x+4x^2)$; $(x+y)^3(x^3+y^3)$.
42. $(x+1)(x^2-x+1)(x^2+x+1)$ or $(x+1)(x^4+x^2+1)$;
 $(x-y)(x^4+x^2y^2+y^4)$.
43. $(x^2-y^2)(x^4+x^2y^2+y^4)$ or x^6-y^6 ; $(x+4)^3(x^2-4x+16)$
 $(x^n-1)(x^n+1)$.
44. $1+x^3+x^4$; $(x^2-y^2)(x^4+x^2y^2+y^4)$ or $(x^3-y^3)(x^3+y^3)$
or x^6-y^6 .
45. $-(a-b)(b-c)(c-a)$; $-(y^2-z^2)(x^2-y^2)(x^2-z^2)$.

EXERCISE LV. (PAGE 145.)

1. $(x+1)(x+2)(x+3)(x+5)(x-5)(x-6)$.
2. $(x+1)(x^2+1)(6x^3+5x^2+2x-1)$.
3. $(x+2)(x+3)(x+4)(x+5)$.
4. $(x+3)(x+4)^2(x+5)$.
5. $3(2x+3)(2x+5)(x^2-x-4)$.
6. $x(x-1)(x-4)(1-2x)(1+2x)$.
7. $6(x-1)^2(x-2)(x+2)(x-4)$.
8. $(x+4)(x^3-2)(4x^3+2x+5)$.
9. $(x+5)(x^2-x+1)(3x^3+5x+6)$.

- 3) $(x-3)(x-12)$.
 $(x-4)$.
 $+b+c)^2$.
1).
3) $(3x-2)$.
 $(3x-1)(3x+2)$.
1) $(4a+1)(5a+1)$.
 $(x-1)(x+3)(3x-2)$.
 $(x-1)(x+1)^2$.
 (x^3-y^3) ; $24(1-x^4)$.
 $y)^3(x^3+y^2)$.
 $+1)(x^4+x^2+1)$;
 $+4)^3(x^2-4x+16)$;
 $(x^3-y^3)(x^3+y^3)$
 $-y^2)(x^2-z^2)$.
145).
-6).
10. $a(a+5)(7a+9b)(4a^2+3a+9)$.
11. $(x-1)(2x+3)(3x-2)(x^2-x+1)$.
12. $(x-1)(x-3)(x+3)(x^2-3)(x^2-2x+3)$.
13. $(3x^2+2x+1)(3x^3-2x^2+2x-1)(2x^5+3x^2-2x+1)$.
14. $(x+1)(x+2)(x+3)(x+4)$.
15. $(x+1)(x-3)(3x^2-4x+6)(3x^2-6x+4)$.

EXERCISE LVI [a]. (PAGE 146.)

1. -30. 2. 20. 3. $a=b=12$. 4. 10.
5. $a=-10$ and $b=-1$. 6. -114. 7. 13.
8. $c=22$, $a=48$. 9. $1-a$. 10. $b=2$. 11. $c=b^2(1-a)$.
13. Divisor $= (x-ay)(x-bz)$, and dividend vanishes for each of these factors; *i.e.*, for $x=ay$, $x=bz$, substitute and subtract.
15. Substitute $x=-a$ in each expression; subtract and $a=p-1$; substitute this in $a^2-qa+1=0$.

[b.]

1. Remainders on dividing by $x+c$ is zero. First quotient $= (x+a-c)$, which multiplied into $x^2+a'x+b'$, gives required expression.
6. Unity. 7. $3x^2+2x+1$.
8. $2x(4x^2+1)(5x^2-1)^2(5x^2+x+1)$.

EXERCISE LVII [a]. (PAGE 153.)

1. $\frac{a+b}{a}$; $\frac{b-a}{b}$; $\frac{x+1}{x}$; $\frac{x-1}{x}$; $\frac{3x+2}{x}$; $\frac{xy+1}{x}$; $\frac{x^2y-1}{x}$.
2. $\frac{x^2+x+1}{x}$; $\frac{a^2-a-1}{a}$; $\frac{a^4+a^3+1}{a^2}$; $\frac{6x^3+4x^2-3}{2x}$.
3. $\frac{2x+1}{x}$; $\frac{2x^2+x-2}{x}$; $\frac{3x^3-x^2+2}{x}$; $\frac{3a^2b^2-1}{ab}$.
4. $\frac{x}{x-1}$; $\frac{3x-5}{x-1}$; $\frac{x^2(4x^3+3)}{x^3+1}$; $\frac{x(x^3+1)}{x^2+1}$.
5. $\frac{(a+b+c)(a+b-c)}{2ab}$; $\frac{(b+c-a)(a-b+c)}{2ab}$; $\frac{x^3+1}{x-1}$.
6. $\frac{x(x+3)(x+5)}{x+2}$; $\frac{x^2-a^2+xy+ay+1}{x+a}$; $\frac{3x^2+2x+1}{x+4}$.

7. $\frac{22a^2 - 40ab + 16b^2}{5a - 6b}$; $\frac{a^3 + x^3 + a^2 - x^2}{a + x}$.
8. $\frac{2x^4 + x^2y^2 + y^4}{x^2 - xy + y^2}$; $\frac{x^2 + xy + y^2}{x + a}$; $\frac{x^3}{x - 1}$.
9. $\frac{3x^4 - 24x^2 + 3a^2}{a^2 + 2ax + x^4}$; $\frac{(a - 2b)(a + 2b)(a^2 + 4b^2)}{(a - 3b)(a + 3b)}$; $\frac{8x^3 - 27}{2x - 7}$.
10. $\frac{a^3 - b^3}{a + b}$; $\frac{x^3 + y^3}{x - y}$; $\frac{x^3 - y^3}{x + y}$;
 $\frac{2x(1 - 3x^2 + 3x - 9x^4 + 9x^5 + 27x^6 - 27x^7)}{1 + 3x}$.

[b.]

1. $a + \frac{1}{a}$; $a - \frac{1}{a}$; $x + \frac{y}{x}$; $1 + \frac{b^2}{a^2}$; $a + \frac{a^2}{b^2}$; $2 + \frac{a}{x + a}$.
2. $1 + \frac{3}{x - 2}$; $1 + \frac{7}{x - 4}$; $1 + \frac{2}{a - 5}$; $1 + \frac{y^2}{x^2 - 2y^2}$;
 $1 - \frac{2}{a^2 + 1}$; $a - 1 - \frac{1}{a + 1}$.
3. $x^2 + x + 1 + \frac{2}{x - 1}$; $x + a - \frac{a^2}{x - a}$; $x - a + \frac{a^2}{x - a}$;
 $1 + x + \frac{x^2}{1 + x}$.
4. $x - \frac{4a^2}{x + 2a}$; $1 + \frac{2}{x^2 - 3x - 1}$; $1 + \frac{a}{2x^2 - 3x + 1}$;
 $x^2 - ax + a^2 - \frac{5a^3}{x + a}$.
5. $1 - \frac{2}{x + 1} + 1 + \frac{2}{x - 1}$; $1 + \frac{1}{x - 4} + 1 + \frac{1}{x - 5}$;
 $1 - \frac{4x^2 + 3x - 7}{x^3 + 4x^2 - 5}$.
6. $1 + \frac{a + b - c - d}{mx + c + d}$; $1 + \frac{b - c}{mx - b - d}$; $1 + \frac{2}{ax + m - 1}$;
 $3x + 1 - \frac{11}{6x - 9}$.
7. $1 + \frac{2}{ax + m - 2} + 1 + \frac{2}{ax + n - 2}$;
 $x + 2 + \frac{1}{x - 8} + x + 2 + \frac{1}{x - 6}$.

$$8. 12x - 25 + \frac{245x - 49}{5x^2 + 9x - 2}; \quad x^2 - xy + y^2 - \frac{2y^3}{x + y};$$

$$a - 6b - c - 1 + \frac{16b^2 + 8bc + b}{a + 2b + c}.$$

$$16b^2); \quad \frac{8x^3 - 27}{2x - 7}.$$

EXERCISE LVIII [a]. (PAGE 156.)

$$1. \frac{a}{b}; \frac{c}{d}; \frac{a^2}{b^2}; \frac{1}{c}; \frac{x}{y}; \frac{ab}{c}; \frac{x^2}{a^2}.$$

$$2. \frac{ax}{b}; \frac{x}{4y^2}; \frac{4b}{5a}; \frac{2x}{3a^2y}; \frac{1}{3abc}.$$

$$3. \frac{1}{3bx}; \frac{x^2}{a^{m-2}}; \frac{y^n}{3x}; \frac{a^{m-2}}{b^{m-2}}; \frac{a-b}{ab}.$$

$$2 + \frac{a}{x+a}.$$

$$\frac{y^2}{x^2 - 2y^2};$$

$$4. \frac{a}{y-x}; \frac{b}{2a+3c}; \frac{x+y}{x-y}; \frac{2a-3b}{3a-2b}; \frac{a}{a-b}.$$

$$5. \frac{x}{x^2-1}; \frac{1}{y(a+x)}; \frac{1}{y}; \frac{1}{a^2-b^2}; \frac{a+b}{a-b}.$$

$$-a + \frac{a^2}{x-a};$$

$$6. \frac{a+b}{a^2+ab+b^2}; \frac{x^2-x+1}{x^2+2x+1}; \frac{a^2+ab+b^2}{a+b}; \frac{x}{x-8}; \frac{x-3}{x-5}.$$

$$7. \frac{x+4}{x+7}; \frac{x+7}{x-3}; \frac{x+7a}{x-3a}; \frac{x+4}{x+3}.$$

$$8. \frac{x^2+x+1}{x^2-2x+1}; \frac{x+b}{x+c}; \frac{x+b}{x-c}.$$

$$9. \frac{3(a+7b)}{4(a+5b)}; \frac{a+b-c}{a+b+c}; \frac{a^2-b}{b^2-a}; \frac{1}{a-b}.$$

$$10. \frac{2x+a+b}{2x-a-b}; \quad y \text{ in numerator should be } b \text{ and in denominator } x;$$

$$\frac{1}{a-b}; \frac{a+b}{a-2b}; \frac{5a^2+3b}{a^2-b}.$$

$$11. \text{Irreducible}; \quad \frac{y+z}{z-y}; \frac{3x+2}{4x+5}; \frac{(a+b)^2}{a-b}.$$

$$12. \frac{x^2-xy+y^2}{x^2-2xy+y^2}; \frac{a+b}{a-b}; \frac{x^2+1}{x^2-1}.$$

[b.]

$$1. \frac{x-11}{4x^2-5x-5}; \text{irreducible}; \quad \frac{x+4}{(x-1)^2}.$$

2. $\frac{7x-2y}{5x^2-3xy+2y^2}$; $\frac{x^2+x-2}{x^2+5x+5}$; $\frac{5a^3(a+x)}{x(a^2+ax+x^2)}$;
in first term of numerator, x should be a .
3. $\frac{9(x^2+y^2+z^2-xy-xz-yz)}{2x-y-z}$; $\frac{ax+by}{ax-by}$.
4. $\frac{x+2}{2x-1}$; $\frac{(x+1)(3x-7)}{(x-1)(7x+3)}$.
5. $\frac{a-b}{1}$; $\frac{(2x-3a)^2}{1}$; $\frac{x^3-2x^2+12x-18}{x^3-2x^2+x+4}$.
6. $\frac{3(x-3a)(x-4a)}{2(x+3a)(x+4a)}$; $\frac{a(x+8a)}{x(x+7a)}$.
7. $\frac{3ax^2+1}{4a^2x^4+2ax^2-1}$; $\frac{c}{1}$.
8. $\frac{a^2+b^2}{a}$; $\frac{x-5}{x+5}$; irreducible.
9. $\frac{a^2+b^2+c^2-ab-bc-ca}{a^2+b^2+c^2+2ab+2bc+2ca}$; $\frac{x+y+z}{2}$.
10. $\frac{1}{8(a+b+c)}$; Expre'n = $\frac{(x-1)(x^{n-1}+x^{n-2}+\dots+x+1)}{(x-1)\{nx^n-(x^{n-1}+x^{n-2}+\dots+x+1)\}}$
 $= \frac{(x-1)(x^{n-1}-1+x^{n-2}-1+\dots+x-1)}{(x-1)(x^n-x^{n-1}+x^n-x^{n-2}+\dots+x^n-1)} =$
 $\frac{(x-1)^2\{x^{n-2}+2x^{n-3}+3x^{n-4}+\dots+(n-1)\}}{(x-1)^2\{x^{n-1}+x^{n-2}(x+1)+x^{n-3}(x^2+x+1)+\dots+(x^{n-2}+2x^{n-3}+3x^{n-4}+\dots+(n-1))\}}$
 $= \frac{x^{n-2}+2x^{n-3}+3x^{n-4}+\dots+(n-1)}{x^{n-1}+x^{n-2}(x+1)+x^{n-3}(x^2+x+1)+\dots+1}$
last numerator
 $= \frac{nx^{n-1}+(n-1)x^{n-2}+(n-2)x^{n-3}+\dots+1}{}$
11. $\frac{(a+b)(b+c)(c+a)}{1}$; $\frac{x+2y+3z}{x-3y-4z}$.
12. $\frac{5xy(x+y)}{2(x^2+xy+y^2)}$; $\frac{a^2+b^2+c^2+ab+bc+ca}{5(a^2+b^2+c^2-ab-bc-ca)}$.

EXERCISE LIX [a]. (PAGE 162.)

1. $\frac{3}{x}$; $\frac{1}{x}$; $\frac{a}{x}$; $\frac{x+y}{a+b}$; $\frac{12}{y}$.
2. $\frac{4(a+b)}{5}$; $\frac{2(a-b)}{7}$; $-\frac{2b}{b} = -2$.

8. $\frac{a+b}{ab}$; $\frac{b-a}{ab}$; $\frac{a^2-x^2}{ax}$; $\frac{a+b}{abx}$; $\frac{a(a+x)}{x^2}$; $\frac{a(a-1)}{x}$.
9. $\frac{5x-17}{6}$; $\frac{x+17}{6}$; $\frac{12a-5}{35}$; $\frac{2a^2b-1}{10b^2}$.
10. $\frac{7x+2}{12}$; $\frac{x(b-1)}{ab}$; $\frac{ab+bc+ca}{abc}$; $\frac{a^2b^2-c^2}{abc}$.
11. $\frac{a+b+c}{abc}$; $\frac{x^2+y^2+z^2}{xyz}$; $\frac{bcx+acy+abz}{abc}$; $\frac{x^2+18x-27}{9x^2}$.
12. $\frac{6a+4b+3c}{12x}$; $\frac{x(az+ay+yz)}{ayz}$; $\frac{a+bx+cx^2}{x^n}$; $\frac{x^2-a^2b^2}{abx}$.
13. $\frac{2x}{x^2-1}$; $\frac{2b}{a^2-b^2}$; $\frac{2x^2}{x^2-a^2}$; $\frac{a^2+x^2}{a^2-x^2}$.
14. $\frac{11}{x^3+3x-28}$; $\frac{c-a}{(a+b)(b+c)}$; $\frac{x+4}{x^2+5x+6}$; $\frac{2a(a-1)}{a^2-b^2}$.
15. $\frac{2(x^2+1)}{x^2-1}$; $\frac{2(x^2+a^2)}{x^2-a^2}$; $\frac{2x^2-32x+127}{x^2-17x+72}$; $\frac{24x}{4x^2-9}$.
16. $\frac{x^2-2xy+y^2}{4y(x+y)}$; $\frac{3(2x+7)}{(x-1)(x-2)(x+4)}$; $\frac{1+a}{x+a}$;
 $\frac{2(1+nx)}{(n^2-1)(1-x^2)}$.
17. $\frac{2x^3}{x^4+x^2+1}$; $\frac{3ax}{x^3+a^3}$; $\frac{3ax}{x^3-a^3}$.
18. $\frac{2a}{a^2-x^2}$; $\frac{2a}{a(a-x)}$; $\frac{1}{a}$.
19. 1 ; $\frac{a}{(1-a)^3}$; $\frac{2(x^2-ax+a^2)}{x^2-a^2}$.
20. $\frac{2x^2-2xy-y^2}{x^2-y^2}$; $\frac{4xy}{(x^2-y^2)^2}$; $\frac{a+b}{a^2-ab+b^2}$; $\frac{a-b}{a^2+ab+b^2}$;
 $\frac{2x-7}{(x-2)(x-3)(x-4)}$; $\frac{2(a+x)}{a^2+ax+x^2}$.

[b.]

1. $\frac{1-6x^2}{1-4x^2}$; $\frac{x}{4x^2-y^2}$.
2. $\frac{4a^2+b^2}{4a^2-9b^2}$; $\frac{4}{a-b}$.
3. $\frac{18}{x^3-27}$; $\frac{2x^2}{x^3+8}$.
4. $\frac{16x^2+29x-104}{12(x-5)(x+5)}$; $\frac{x+3}{x^4-1}$.

5. $\frac{x+11}{(x-1)(x+4)}$; $\frac{2x^3-5x^2y+10xy^2+5y^3}{(x^2-y^2)^3}$.
6. $\frac{a+b+c}{(a+b-c)(a+c-b)(b+c-a)}$; $\frac{6}{(x-1)(x+1)(x+3)}$.
7. $\frac{-1}{x+3y}$; $\frac{2916}{x^3-6561}$. 8. $\frac{8a^7}{x^3-a^3}$; $\frac{a^2}{(c-b)(a-c)}$.
9. $\frac{1}{a+b}$. 10. $\frac{1}{x+2}$; $\frac{4(ab-cd)}{a^2+c^2-b^2-d^2+2ac-2bd}$.
11. 1; 0. 12. $\frac{a}{xyz}$; $\frac{x(1-3x+3x^2)}{(1-x)^3}$.
13. $\frac{1+2x+3x^2}{4(1-x^4)}$; $\frac{x+c}{(x-a)(x-b)}$.
14. $\frac{4a^mb^m}{a^{2m}-b^{2m}}$; $\frac{y^{n-1}z}{(y-z)^n}$.
15. $\frac{a^3b+a^2c+a^2d-c}{a^2}$; $\frac{8x^2+4x-3}{(x-1)(x+1)(2x+1)}$.
16. $\frac{a(a^2+2ax+3x^2)}{4(a^4-x^4)}$; $\frac{2x}{x+y}$.
17. $\frac{2+x^2-x^3}{2(x^2+1)(x^3+1)}$; 2. 18. $\frac{1}{(x+1)(x+2)(x+3)}$.

[c.]

1. $\frac{4a^2-2ab}{a^2-b^2}$, ab for a^2b in num. and denom.; $\frac{b^2(a^3-a^2+b^3)}{a^6-b^6}$.
2. $\frac{16x^{16}}{1-x^{16}}$; $\frac{q(b-a)}{(x-a)(x-b)}$.
3. $\frac{(1-x)(1+2a+a^2)}{(1-a)(1+a)(1+a^2)}$; $\frac{p(a-c)(x-b)+q(b-c)(x-a)}{(x-a)(x-b)(x-c)}$.
4. $\frac{24(x-9)}{(x-1)(x-3)(x-5)(x-7)}$;
 $\frac{-22x^3+44x^2-180x-622}{7(x-1)(x+2)(x-3)(x+4)}$.
5. 0. 6. 0. 7. 1. 8. $a+b+c$.
9. 0. 10. 0. 11. 0. 12. 0. 13. 0. 14. 0. 15. 1.

16. $(x^3 - y^3)$; sign + should be - between given quantities.
17.
$$\frac{a^2b(a^{2x} + b^2)}{(a^{2x} - a^2b + b^2)(a^{4x} - a^{3x} + a^{2x}b^2 - a^2b^3 + b^4)}.$$
18.
$$\frac{a^{4x}\left(a^{2y} - \frac{1}{a^{2y}}\right) + a^{2x}\left(a^{2y} + \frac{1}{a^{2y}}\right) - (a^{4x} + 1)}{1 - a^{4x}}.$$

EXERCISE LX [a]. (PAGE 170.)

1. $a^2 \div b^2$; $a^2 \div c^2$; $b \div a$; $\frac{x^4}{a^4}$; $b^3 \div a^2$; $b + x$.
2. $(a + b) \div (a - b)$; $4a^3 \div 15x^6$; $\frac{5}{3}a^2x$; 1.
3. $\frac{a^2 - 9}{a^2 - 16}$; 1; $\frac{x - y}{x^2 + y^2}$; $(x + 3)(x + 1)$; $1 \div x$.
4. $\frac{1}{(x + 4)^2}$; 1; $-\frac{a}{b}$; $\frac{1}{z} - \frac{1}{x}$; $\frac{5b}{3a^{2m}c}$.
5. $xy + yz + zx$; $2(a + b + c)$; $ab + bc + ca$.
6. $2 + 2x^2$; $4ab$; $\frac{1}{a(x - a)}$; $\frac{n}{m}$.
7. $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$; $\frac{2(a^2 + b^2)}{(a^2 - b^2)^2}$; 1; $\frac{1 - d}{1 + b}$.
8. $-2x$; $\frac{x^2}{y^2}$; $\frac{a^{3x-1}b^{4y-1}}{c^{x+1}}$.
9. $\frac{ax}{(x - b)^2}$; $\frac{x}{x - 3}$; $x - 1$; $(x - a)^6 \div a^2$.
10. $\left(\frac{x + 3}{x - 3}\right)^3$; $\frac{a}{b}$; $\frac{a(a^2 + ax + x^2)}{(a - x)(a + x)^2}$.
11. $\frac{x^2 + xy + y^2}{x + y}$; $\frac{1}{x^3 + a^3}$; $\frac{a^2 - ab + b^2}{(a + b)(a - b)^2}$; $\frac{1 - b^n + b^{2n}}{(1 + b^n)(1 - b^n)^2}$.
12. $\frac{(x^2 - 1)(x + 1)}{x^2 + x + 1}$; 1; $\frac{(x + b)(x - c)}{(x + a)(x + c)}$.
13. $\frac{x - 3b}{x - 2c}$; $\frac{(x - c)^2}{(x + a)(x - b)(x + b)}$.
14. $\frac{x + b}{x + c}$; $(x + 2)$; $\frac{1}{x^2} - 1$.

[b.]

1. $\frac{1}{(x-y)^2}$; $\left(\frac{5ab^2}{2}\right)^m$.
2. 1 ; $\frac{(x^2-1)(x+1)}{(x-7)(x-5)^2}$.
3. $\frac{x+a}{x-a}$; $\frac{a}{b}$.
4. $\frac{a-b}{a+b}$; $\frac{2ab^6}{15cd^6}$.
5. $\frac{x^2(x-1)}{x^2-4}$; $(100xy)^m$; $\frac{a^2}{b^2} + \frac{b^2}{a^2} - 1$.
6. $\frac{a^2(x+13a)}{b^2c(x-2a)}$, put x for a in first term of numerator and a for a^2 in second term; $\frac{625a^6}{864b^2}$.
7. $\frac{a+b-c}{b-c-a}$; 1 .
8. $\frac{x^4}{x^2+y^2}$; $\left(\frac{5bc^3d^2e}{3a^2f^6}\right)^m$.
9. $\frac{(x+a)(x^3+a^3)}{(x^2+a^2)^2(x^2+ax+a^2)}$; $\frac{1}{(x-4)(3x-2)}$.
10. $\frac{1-y}{x}$; 1 .
11. $\frac{x^2-6x-8}{x-12}$; 1 .
12. $\left(\frac{x+y}{x-y}\right)^3$; $\frac{c-d}{(a+b)^4}$.
13. $\frac{(x-a)^2(x+3a)}{(x-c)^2(x+6a)}$; $\frac{(5a^{2m})^x}{7^{mx-1}}$.
14. $abcde$; $\frac{y^6}{(y-x)(x^2+y^2)}$.
15. 2 ; 0 .
16. 1 ; $\frac{675cd}{8ab}$.
17. $\frac{abc}{(a-c+b)(ab+bc+ca)}$.
18. $\frac{ax^3(x^2+a^2)}{x^3+a^3}$; $-\frac{p^4q^4}{p^4+p^2q^2+q^4}$.

EXERCISE LXI [a]. (PAGE 176.)

1. $\frac{4a-3}{16}$; $\frac{8a+1}{28}$; $\frac{x-3}{6}$; $\frac{a^2-bc}{b^2}$; $\frac{a^2}{b^4}$; ab .
2. $\frac{2(a-b)}{3}$; $\frac{16+x}{16-x}$; $\frac{6x-20}{12x+5}$; $\frac{2x-5}{x-10}$; $\frac{12x-8}{12x+9}$.
3. $-\frac{1}{b}$; $\frac{a+b}{a-b}$; 1 ; $\frac{b+a}{b-a}$; $\frac{x}{2+x}$.
4. $\frac{a^2}{b^2}$; $\frac{24b+9a}{12a+8b}$; $\frac{x^2-1}{x^2+1}$; 1 .

5. $\frac{a-ax-1}{a-ax+1}$; $\frac{1}{a}$; $\frac{2ax}{a^2+x^2}$; $\frac{2x-35}{x^2}$.
6. $\frac{x^3-2x^2-3}{x^3(3-x)(3+x)}$; $\frac{1}{x}$; $a+x$.
7. $\frac{a^3+x^3-a^2}{a^3+x^3-x^2}$; $\frac{12}{(x-1)(x-2)}$; $-\frac{a}{x}$.
8. $\frac{(a-b)^2}{a^2}$; $(a+1)^2$; a^2-ax+x^2 .
9. $\frac{2ab}{a^2+b^2}$; $\frac{4ab(a^2+b^2)}{a^4+6a^2b^2+b^4}$; $\frac{(a^2+b^2)(a^4+14a^2b^2+b^4)}{2ab(3a^4+20a^2b^2+3b^4)}$.
10. $\frac{b-a+4}{b-a+5}$; $\frac{6a}{1-2x}$.
11. $\frac{a^2+b^2}{b-a}$; $\frac{ab+bc+ca}{a^2c+ab^2+bc^2}$.

[b.]

1. $\frac{4ab(a+b)}{(a-b)(a^3+2a^2b+ab^2-a^2-b^2)}$; $\frac{4(ax^2-4)}{x^3(3x-2)(3x+2)}$; $\frac{1+x}{1+x^2}$.
2. $\frac{1}{(x^2-y^2-z^2)(x^2+y^2-z^2)}$; $\frac{2}{63}$; $\frac{a-3}{(a-1)(a+3)}$.
3. $\frac{b^2-ab}{a^2+ab}$; x^2y^2 .
4. $\frac{1}{x^3+1}$; $\frac{x(x^2+2)}{x^4+3x^2+1}$.
5. 1 ; $\frac{1}{a^2b(b-a)}$.
6. 0 ; $\frac{x+2y+6}{8x(y+6)}$.
7. $-x$; $\frac{1+2x}{2+3x}$.
8. $\frac{a^3}{(a+x)(a^2-x^2)}$; $\frac{1}{x^3}$.
9. $\frac{4a^2b^2}{(a^2-b^2)^2}$; 1 .

[c.]

1. 1 ; $a+b$.
2. $16x+11y$.
3. $5x+11y$.
4. $\frac{x-y}{x+y}$; $\frac{3-3x}{4+4x}$.
5. 1 .
6. $\frac{2}{x}$.

7. $\frac{(a-b)}{(a+b)}$. 8. $\frac{a}{2x^2}$. 9. $\frac{ab}{a+b}$.
 10. 2. 11. $-\frac{1}{2x}$ or $-\frac{2}{x}$. 12. c .

16. Take the fractions in pairs, thus :

$$\left(\frac{1}{s-a} + \frac{1}{s-b}\right) + \left(\frac{1}{s-c} - \frac{1}{s}\right) = \frac{c}{(s-a)(s-b)} + \frac{c}{s(s-c)},$$

by substituting for $2s$, etc.

17. a . 20. Multiply given relations out and transpose,

$$a + b + c + d = abc + abd + bcd + acd = abcd \left(\frac{1}{a} + \text{etc.}\right) \text{ etc.}$$

EXERCISE LXII. (PAGE 185.)

21. 1. $1 + x + x^2 + x^3 + x^4 + \dots$
 2. $1 + 3x + 9x^2 + 27x^3 + 81x^4 + \dots$
 3. $1 - x + x^2 - x^3 + x^4 - \dots$
 4. $1 - 3x + 9x^2 - 27x^3 + 81x^4 - \dots$
 5. $\frac{a^2}{x} + \frac{a^2b}{x^2} + \frac{a^2b^2}{x^3} + \frac{a^2b^3}{x^4} + \frac{a^2b^4}{x^5} + \dots$
 6. $x + \frac{x^2}{a} + \frac{x^3}{a^2} + \frac{x^4}{a^3} + \frac{x^5}{a^4} + \dots$
 7. $a + abx + ab^2x^2 + ab^3x^3 + ab^4x^4 + \dots$
 8. $1 - \frac{2x}{a} + \frac{3x^2}{a^2} - \frac{4x^3}{a^3} + \frac{5x^4}{a^4} - \dots$
 9. $1 + x - x^2 - x^3 + x^4 + \dots$
 10. $1 + ax + a^2x^2 + a^3x^3 + a^4x^4 + \dots$
 27. $\frac{x^3 + y^3}{x^3 - y^3} = \frac{(x+y)^3 - 3xy(x+y)}{(x-y)^3 + 3xy(x-y)} = \text{etc.}$
 Substitute from given conditions.

EXERCISE LXIII [a]. (PAGE 191.)

1. $x = 7$. 2. $x = 1\frac{1}{2}$. 3. $x = 2$.
 4. $x = 3$. 5. $x = 4$. 6. $x = \frac{2}{11}$, read x for $2x$.
 7. $x = \frac{1}{2}$. 8. $x = 5$. 9. $x = 6$.
 10. $x = \frac{a(1+b)}{a-2}$. 11. $x = \frac{3a-b}{2}$. 12. $x = 1$.

$\frac{c}{s(s-c)}$,
etc.) etc.

13. $x = \frac{a^2(b-a)}{b(a+b)}$. 14. $x = \frac{3a-6}{4}$. 15. $x = \frac{2ab}{a+b}$.
16. $x = \frac{d(a+c)}{b}$. 17. $x = b$. 18. $x = 9$.
19. $x = -\frac{3}{4}$. 20. $x = 3$. 21. $x = 60$.
22. $x = 1$. 23. $x = -\frac{11}{2}$. 24. $x = -\frac{2}{7}$.
25. $x = \frac{ab}{a+b-c}$. 26. $x = \frac{a^2}{b-a}$. 27. $x = -6\frac{1}{4}$.
28. $x = -\frac{7}{18}$. 29. $x = -\frac{3}{2}$. 30. $x = 3a$.
31. $x = \frac{bn+dm+amn+cmn}{b+d+am+cn}$. 32. $x = 14$.
33. $x = \frac{acn-abn-abm-bcm}{nb-nc-ma-mc}$. 34. $x=3a$; right mem. sh. be $2\frac{1}{4}$.
35. $x = 7$. 36. $x = 3$. 37. $x = 3$.
38. $x = \frac{1}{4}$. 39. $x = 11$. 40. $x = -6$.

[b.]

1. $x = -3\frac{1}{4}$. 2. $x = -107$. 3. $x = \frac{a+b+c+d}{n-m}$.
4. $x = 3$. 5. $x = \frac{b(a-b+c)}{a}$. 6. $x = \frac{1}{ab}$.
7. $x = 0$ or $4\frac{1}{8}$. 8. $x = -\frac{7}{16}$. 9. $x = \frac{m}{c}$.
10. $x = 15$. 11. $x = 3$. 12. $x = 1\frac{1}{2}$.
13. $x = -2\frac{1}{2}$. 14. $x = -6$. 15. $x = 7$.
16. $x = \frac{m}{2}$. 17. $x = \pm 3$ or ∞ .
18. $x-1=0$ and $4x^2+5x+3=0$. 19. $x = a+b$.
20. $x = 2$. 21. $x = a$. 22. $x = \frac{7bc}{9b+4c}$.

[c.]

1. $x = 13$; second numerator should be 3. 2. $x = -9$.
3. $(x-3)(2x-5) = 0$; $x = 3$.
4. $x = 4\frac{1}{2}$. 5. $x = -1$.

$5x^2 - 16x - 83 = 0$

for $2x$.

$$6. x = \frac{ab - bc - ca}{c^2}.$$

$$7. x = \frac{a(a-b)(a-c)}{(a+b)(a+c)}.$$

$$8. x = 0.$$

$$9. x = 5.$$

10. $x(b-a) = 0$; whence $x = 0$, unless $b-a = 0$, in that case x may have any finite value.

$$11. x = \pm \sqrt{5} \text{ or } 0.$$

$$12. x = c. \quad x^2 + 2x(a-c) + 2a^2 - 2ad - c^2 = 0$$

$$13. x(2x+5) = 0; \therefore x = 0 \text{ or } -2\frac{1}{2}.$$

$$14. x = a + b + c.$$

$$15. x = a^2 + b^2 + c^2.$$

$$16. x = a^4. \quad (\text{First numerator on right hand should be } x-1.)$$

17. Take in pairs the fractions with like numerators;

$$x = \frac{np(c-a) + mp(a-b) + mn(b-c)}{m(a-c) + n(b-a) + p(c-b)}.$$

$$18. (-7x+49) \left\{ \frac{1}{x^2+x-2} - \frac{1}{x^2+x-12} \right\} = 0;$$

$$\therefore x = 7 \text{ or } \infty.$$

19. Complete the divisions, cancel and transpose;

$$\therefore \frac{2}{x-4} - \frac{1}{x-5} - \frac{1}{x-2} = 0;$$

$$\text{or } \frac{1}{x-4} - \frac{1}{x-2} = \frac{1}{x-5} - \frac{1}{x-4};$$

$$\text{whence } (x-8)(x-4) = 0; \therefore x = 8.$$

The value 4 is not admissible.

$$20. x = \frac{bn(q-p)(m-p) + ap(q-n)(m-n)}{b(q-p)(m-p) + a(q-n)(m-n)}.$$

$$21. x = \{m(b-c) - n(a+c)\} \div (m-n)$$

$$22. (a^2x + b^2x - a^2b - ab^2 - b^2c + a^2c) \times \left\{ \frac{1}{(x-a)(x-b)} - \frac{1}{(x-a-c)(x-b+c)} \right\} = 0.$$

$$\therefore x = \{a^2(b-c) + b^2(c+a)\} \div (a^2 + b^2).$$

$$23. (a-b) \left(\frac{x}{n-o} - \frac{1}{p-q} \right) = 0, \quad x = \frac{n-o}{p-q}.$$

$$24. \frac{x-2a}{b+c-a} - 1 + \text{anal.} + \text{anal.} = 0, \text{ whence}$$

$$(x-a-b-c) \left\{ \frac{1}{b+c-a} + \text{anal.} + \text{anal.} \right\} = 0;$$

$$x = a + b + c.$$

25. $\frac{a-x}{a^2-bc} - \frac{1}{a+b+c} + \&c. + \&c. = 0$, or
 $\frac{ab+bc+ca-(a+b+c)x}{(a^2-bc)(a+b+c)} + \text{anal.} + \text{anal.} = 0$;
 $x = (ab+bc+ca) \div (a+b+c)$.

EXERCISE LXIV [a]. (PAGE 198.)

PROBLEMS.

1. $10\frac{5}{12}$ dozen. 2. \$36000. 3. 12 years.
 4. \$300. 5. $\frac{ma-12b}{12-m}$. 6. $\frac{abc}{b+c}$.
 7. 754. 8. \$3.75. 9. 142857.
 10. \$8000. 11. $190\frac{50}{121}$ bushels. 12. 90 and 91.
 13. Equation reduces to $(4-4)x + 40 = 0$; $x = \infty$; *i. e.*, conditions of problems are inconsistent. In fact, *area* will *always* be 45 ft. less, under the given conditions; for using 45 for 85, the resulting equation is an identity.
 14. \$1857.35 $\frac{3}{4}$ and \$142.64 $\frac{2}{3}$.
 15. 857142; x representing number, equation is
 $\frac{1}{10}(x-2) + 200,000 = \frac{1}{3}x$. 16. \$7.60.
 17. 550; read 6 in first line; 4 times and 6 cts. in second line.
 18. $13\frac{1}{2}$ feet and $16\frac{1}{2}$ feet.
 19. A, \$2800; B, \$3900; C, \$5138; D, \$2196; E, \$2966.
 20. \$14.

[b.]

1. 960 gallons. 2. 420 acres. 3. \$1280, $7\frac{1}{2}\%$.
 4. Gain or loss % = $\frac{q(100+n) \sim 100p}{p}$, according as
 $q > \frac{100p}{100+n}$
 $q < \frac{100p}{100+n}$
 5. B makes 1740 yds. in 4 m. 34 sec.; C makes 1700 yds. in 4 m. 32 sec. Let x = time in min. from starting at which A overtakes B, then $\frac{x}{4\frac{1}{2}} \cdot 1760 = 20 + \frac{x}{4\frac{1}{2}} \cdot 1740$, $x = 1\frac{1}{2}$ min., distance $775\frac{1}{2}$ yds. from start. Similarly A is found to pass C in $3\frac{1}{2}$ m.; distance $1456\frac{1}{2}$ yds. from start.

6. $10\frac{25}{121}$ miles. 7. 5 gal. 8. \$7400. 9. 57 miles.
 10. $mnpqr \div (mnpq - mpq - npq - mnq - mnp)$.
 11. 484. 12. 1, 2, and 3.
 13. 12000 sq. yds.; 45 cts. 14. 189.
 15. $x = \text{distance}$; then $\frac{2x}{b} = \frac{x}{ap} + \frac{px-x}{bp}$.
 16. Let $2x$ and x be digits; $(2001x)^2 - (1002x)^2 = 2999997x^2$
 $= 4x^2 \times 749,999\frac{1}{2}$. 17. 180,000.
 18. $\left(\frac{n+1}{n}\right)^{n+1}$. 19. $\frac{p(11m-21n)}{20(m-n)}$.
 20. Regular rate 40 miles, diminished rate $38\frac{2}{3}$ miles; 100 miles.
 21. $221 : 273 :: 187 : 231$. 22. $\frac{ap-an}{m-n}$. 23. $\frac{1}{2}$. 24. 14172.

EXERCISE LXVII. (PAGE 212.)

1. $x=3$; $y=2$. 2. $x=6$; $y=-4$.
 3. $x=1$; $y=3$. 4. $x=5$; $y=4$.
 5. $x=4$; $y=-3$. 6. $x=-2$; $y=3$.
 7. $x=1$; $y=2$. 8. $x=-20\frac{1}{2}$; $y=-19\frac{1}{2}$.
 9. $x=10$; $y=9$. 10. $x=\frac{541}{112}$; $y=\frac{1111}{112}$.

EXERCISE LXVIII. (PAGE 213.)

1. $x=2$; $y=3$. 2. $x=\frac{19}{4}$; $y=-\frac{1}{4}$.
 3. $x=3$; $y=-2$. 4. $x=5$; $y=-5$.
 5. $x=4$; $y=4$. 6. $x=\frac{232}{71}$; $y=-\frac{11}{71}$.
 7. $x=3$; $y=2$. 8. $x=2$; $y=1$.
 9. $x=4$; $y=-3$. 10. $x=3\frac{1}{2}$; $y=\frac{1}{2}$.

EXERCISE LXIX. (PAGE 214.)

1. $x=5$; $y=7$. 2. $x=1$; $y=-1$.
 3. $x=2$; $y=-3$. 4. $x=-2\frac{1}{2}$; $y=5\frac{1}{2}$.
 5. $x=5$; $y=-4$. 6. $x=\frac{4}{3}$; $y=\frac{1}{3}$.
 7. $x=1\frac{41}{105}$; $y=2\frac{23}{105}$. 8. $x=5$; $y=6$.
 9. $x=1\frac{33}{105}$; $y=-1\frac{59}{105}$. 10. $x=3\frac{1}{5}$; $y=-3\frac{1}{5}$.

EXERCISE LXX. (PAGE 216.)

1. $x = \frac{a+b}{2}$; $y = \frac{a-b}{2}$.
2. $x = \frac{1-b^2}{a-b}$; $y = \frac{ab-1}{a-b}$.
3. $x = \frac{mp-nq}{m^2-n^2}$; $y = \frac{np-mq}{m^2-n^2}$.
4. $x = 1$; $y = 1$.
5. $x = a+b$; $y = -1$.
6. $x = \frac{b(4b^2-7a^2)}{4b^2-3a^2}$; $y = \frac{a(9a^2-4b^2)}{3a^2-4b^2}$.
7. $x = y = \frac{2}{a+b}$.
8. $x = m+n$; $y = m-n$.
9. $x = y = \frac{m}{a+c}$.
10. $x = \frac{a(bc-2ac-c^2-a^2+3ab)}{2(2ab+bc-ac-c^2)}$; $y = \frac{a(3a^2+ab+4ac+ac^2-bc)}{2(2ab+bc-ac-c^2)}$.

EXERCISE LXXI [a]. (PAGE 218.)

1. $x = 6$; $y = 12$.
2. $x = 4$; $y = 3$.
3. $x = 7$; $y = 10$.
4. $x = 13\frac{2}{9}$; $y = -41\frac{7}{9}$.
5. $x = 6$; $y = 12$.
6. $x = 4\frac{5}{8}$; $y = -12$.
7. $x = 8$; $y = -\frac{1}{2}$.
8. $x = 2$; $y = 7$.
9. $x = y = 5$.
10. $x = 4$; $y = 5$.
11. $x = 8\frac{1}{2}$; $y = -\frac{1}{2}$.
12. $x = y = \frac{a^2b^2}{a^2+b^2}$.
13. $x = \frac{2mn(n^2-m^2)}{n^4+6m^2n^2+m^4}$; $y = \frac{n^4-m^4}{n^4+6m^2n^2+m^4}$.
14. $x = \frac{a^2}{a-b}$; $y = \frac{b^2}{b-a}$.
15. $x = 4$; $y = 1$.
16. $x = 6$; $y = -2$.
17. $x = -2$; $y = 4$.
18. $x = 3$; $y = -4$.

[b.]

1. $x = y = \frac{1}{16}$.
2. $x = 14$; $y = -14$.
3. $x = 4\frac{1}{81}$; $y = 11\frac{7}{81}$.
4. $x = 21$; $y = 20$.
5. $x = 3$; $y = 7$.

6. $x = \frac{ab-1}{(1-a)(1-b)}; y = \frac{a-b}{(1-a)(1-b)}.$
 7. $x = \frac{ab+4b-2a}{ab-b+2a}; y = \frac{a^2+2a+ac-ab+b-c}{ab-b+2a}.$
 8. $x = \pm 9; y = \pm 3.$ 9. $x = a; y = b.$
 10. $x = 9; y = 2.$ 11. $x = \frac{a}{a-b}; y = \frac{b}{a+b}.$
 12. $x = 8; y = 2.$
 13. $x = \frac{b+c-a-d}{4(bc-ad)}; y = \frac{c^2-a-b}{4(ad)}.$
 14. $x = -2\frac{3}{4}; y = -6\frac{1}{4}.$ 15. $x = y = 3\frac{1}{2}.$
 16. $x = 6\frac{2}{3}; y = 8.$
 17. $x = \frac{b'e-bc'}{ab'-a'b}; y = \frac{ac'-a'e}{ab'-a'b}.$

1. $x = y = \infty$, and the equations are *inconsistent*; thus, put $\frac{a}{a'} = \frac{b}{b'} = k$, and $\therefore a = ka', b = kb'$, and substituting these

values of a and b in $ax + by = c$, we get $a'x + b'y = \frac{c}{k}$, which is inconsistent with the second given equation.

2. $x = y = \frac{c}{b}$, *i. e.* the equations are *not independent*; thus, put $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = m$. Then $a = ma', b = mb', c = mc'$, and substituting in (1), we get $ma'x + mb'y = mc'$, which is a *multiple* of the second given equation.

EXERCISE LXXII. (PAGE 222.)

1. $x = 3; y = 6.$ 2. $x = \frac{1}{2}; y = 1.$
 3. $x = \frac{1}{2}; y = 1.$ 4. $x = -2; y = \frac{3}{2}.$
 5. $x = -\frac{2}{3}; y = \frac{1}{3}.$ 6. $x = -\frac{1}{2}; y = 1.$
 7. $x = y = a + b.$ 8. $x = \frac{b(a^2-bc)}{a-b}; y = \frac{c(a^2-bc)}{a-c}.$
 9. $x = -\frac{1}{14}; y = \frac{1}{14}.$ 10. $x = y = \frac{a^2+b^2}{ab}.$

EXERCISE LXXIII [a]. (PAGE 223.)

1. $x = y = z = 4$.
2. $x = 1\frac{5}{8}$; $y = 7\frac{1}{8}$; $z = -2\frac{3}{8}$.
3. $x = 14\frac{1}{2}$; $y = -1\frac{1}{2}$; $z = -3\frac{1}{2}$.
4. $x = 2$; $y = -3$; $z = -4$.
5. $x = 18$; $y = -122$; $z = -9$.
6. $x = 4$; $y = 3$; $z = 7$.
7. $x = 4$; $y = 5$; $z = 6$.
8. $x = -1$; $y = -2$; $z = 5$.
9. $x = 1$; $y = 2$; $z = 3$.
10. $z = -1\frac{3}{4}$.
11. $x = 3$; $y = 21$; $z = -3$.
12. $x = 3$.
13. $x = 6$; $y = 8$; $z = 10$.
14. $x = 6$; $y = -2$; $z = -3$.
15. $x = 3$; $y = 6$; $z = 8$.
16. $x = -5$; $y = 9$; $z = -8$.
17. $x = -4\frac{1}{3}$, $y = -4\frac{2}{3}$, $z = -3\frac{1}{108}$.

[b.]

1. $x = \frac{1}{2}$; $y = \frac{1}{3}$; $z = -\frac{1}{6}$. Divide through by xyz in each equation.
2. $x = 5$; $y = 1\frac{1}{2}$; $z = \frac{5}{18}$.
3. $x = 1\frac{7}{8}$; $y = -\frac{3}{8}$; $z = 2\frac{3}{8}$.
4. $x = \frac{1}{a}$; $y = \frac{1}{b}$; $z = \frac{1}{c}$.
5. $x = \frac{ab + ca - bc}{2a}$; y and z symmetrically.
6. $z = 3abc \div (c - a)(b + c)$, x and y symmetrically.
7. $x = a - b$; $y = b - c$; $z = c - a$.
8. $x = 2a \div (b + c - a)$; y and z symmetrically.
9. $x = 1 \div (a - b)(a - c)$; y and z symmetrically.
10. $z = \frac{5}{11}\frac{9}{8}$.
11. $x = \frac{1}{2}$; $y = \frac{1}{3}$; $z = \frac{1}{6}$.
12. $x = a^2 - b^2$; $y = b^2 - c^2$; $z = c^2 - a^2$.
13. $z = \frac{1}{2}(a + b + c)(2a + b + c)$; x and y by symmetry.
14. $z = \frac{mbpnd + mapnf}{mbp + mna - npc}$; $x = \frac{pmn(acb + bdc)}{mnpd + npfc - pmfb}$.
15. $(x + y + z)(a + b + c) = (a + b + c)^3$; $x + y + z = a + b + c$;

$$x = \frac{a^3 + b^3 + c^3 - (a + b)(b + c)(c + a) + 5abc}{a^2 + b^2 + c^2 - ab - bc - ca}$$

EXERCISE LXXIV [a]. (PAGE 226.)

1. $x = 4$; $y = 9$; $z = 16$; $u = 25$.
2. $x = 3$; $y = 0$; $z = 5$; $u = 2$.
3. $x = 5$; $y = 3$; $z = 1$; $u = 4$.
4. $x = -5\frac{2}{11}$; $y = 16\frac{5}{11}$; $z = -5\frac{3}{11}$; $u = 11\frac{1}{11}$.
5. $x = 3$; $y = 2$; $z = -4$; $u = 5$.
6. $x = 0$; $y = -1$; $z = 2$; $v = -4$.
7. $x = -1$; $y = -2$; $z = -3$; $u = -4$; $t = -5$.
8. $x = 1$; $y = 2$; $z = 3$; $u = 4$.

[b.]

1. $x = a + b + c$; $y = a + b - c$; $z = a - b + c$; $t = b + c - a$.
2. $x = 6$; $y = -1$; $z = 3$; $u = 2$.
3. $x = \frac{1}{3}(a - 2b + c + d)$; y, z , and u by symmetry.
4. $x = \frac{1}{3}(a - b + c - d + e)$; y, z, u , and v by symmetry.
5. $x = \frac{1}{3}(2a - b - c - e + 2d)$; y, z , and u by symmetry.
6. $x = \frac{1}{3}(a + d)$; y, z, u , and v by symmetry.
7. $x = 30$; $y = 20$; $z = 42$; $u = 72$; (y should be z in second equation.)
8. $x = a + b + c$; y, z , &c., by symmetry.
9. $x = \frac{1}{3}(a + b + c + d + e - 4f)$; y, z , &c., by symmetry.
10. Divide each side of every equation by xyz ; $x = 1 \div b - c$; y and z by symmetry.

EXERCISE LXXV [a]. (PAGE 229.)

1. $x = 714285$; $y = 142857$.
2. $x = 40$; $y = 65$.
3. Willie 4; Charlie 8.
4. $x = 1.234$; $y = 5.678$.
5. $x = 147$; $y = 63$.
6. 76.
7. 13 : 17.
8. 73.
9. 480 gallons; 400 gallons; 560 gallons.
10. \$10260; \$7560.
11. $\frac{4}{3}$.
12. $10x + y = 6(x + y)$; $\therefore 4x = 5y$; $\therefore 10y + x = 9x = 5x + 5y$, etc.
13. 98 or 89.
14. A, 200 lbs.; B, 250 lbs.; C, 350 lbs.

15. 82 apples; gave away 2. 16. \$5000; \$3000; \$4000.
 17. 40; 88; 104. 18. 486.
 19. $\frac{a(b-c)}{b-a}$; $\frac{b(c-a)}{b-a}$. 20. $x = \frac{2b}{1-a}$; $y = \frac{2b}{1+a}$.
 21. $\frac{3}{4}$, $\frac{1}{4}$.
 22. A, 105; B, $52\frac{1}{2}$; C, 210 minutes; A, B, and C in 30 minutes.

[b.]

1. First, 220 gallons; Second, 100 gallons.
 2. 3674. 3. A, \$40; B, \$24; C, \$16.
 4. $d(m+n) \div 2mn$; $d(n-m) \div 2mn$.
 5. $x+y : x-y : xy :: 5 : 1 : 18$; $x=9$; $y=6$.
 6. $x = \frac{l(m-1)(q-mr) - (1-mn)(mp-lq)}{m^2l(1-n) + mn(1-m) - m(1-mn)}$.
 7. 130. 8. 315 miles. 9. 9; $8\frac{1}{11}$ miles per hour.
 10. $x = (p+1)n$; $y = (pq-1)n$; $z = (q+1)n$.
 11. $x = 1\frac{1}{2}$; $y = 2\frac{2}{3}$; $z = -12$.
 12. $\frac{c(a+b)}{2ab}$; $\frac{c(b-a)}{2ab}$. 13. $\frac{qrm}{qr-ps}$ hours.
 14. A, \$2.60; B, \$1.26 $\frac{2}{3}$; C, 61 $\frac{1}{2}$ cts. 15. Ans. $3\frac{1}{2}$ miles.
 16. 3000 ft. from first station; x = distance from first station;
 y = A's rate per second; z = B's rate;
 then $\frac{4000}{y} - \frac{4000}{z} = 40$; $\frac{x}{y} - \frac{x}{z} = 30$;
 substitute in this the value of $\frac{1}{y} - \frac{1}{z}$, viz. $\frac{1}{160}$, from first
 equation and $x = 3000$.
 17. Gold coin, \$2; silver, \$1.
 18. $11\frac{1}{2}$ miles; 7 miles; and $5\frac{1}{2}$ miles. 19. \$5200; \$2480.
 20. x = time for A, y for B, z for C;
 then $\frac{1}{y} + \frac{1}{z} = \frac{m}{x}$, or $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{m}{x} + \frac{1}{x} = \frac{m+1}{x}$;
 $\therefore m+1 = \frac{xy+yz+zx}{yz}$, similarly for $n+1$ and $p+1$.

21. x = rate of locomotive, y = rate of coach, z = distance;
 then $6x + 8y = z = 5\frac{1}{2}(x + 1\frac{3}{4}) + 7\frac{1}{2}(y + 1\frac{3}{4})$
 $= 7\frac{1}{2}x - 1\frac{3}{4} + 5\frac{1}{2}y - 1\frac{3}{4}$; $x = 38\frac{1}{2}$, $y = 7$, $z = 287$.
After should be *before* in first line of equation.

22. Let m , n be the required *dividends*;

$$\text{then given fraction} = \frac{m}{3 + 4z} + \frac{n}{6 + 7z}.$$

Multiply out and equate coefficients, and we get

$$6m + 3n = 27, \quad 7m + 4n = 34; \quad m = 2, \quad n = 3.$$

$$23. \frac{bc-ad}{cf-de}; \frac{ab-be}{cf-de}. \quad 24. \frac{9}{8x-7}; \frac{6}{5x-4}; \frac{3}{2x-1}.$$

25. 16, 20, 42.

EXERCISE LXXVI. (PAGE 239.)

6. $-\frac{2}{3}$, -10 ; $\pm(a-b)$; $\pm(1-a)$; $\pm a + \frac{2}{3}$.
 7. $\pm(3a-2b)$; $\pm \frac{a+b}{a-b}$; $3, \frac{2}{3}$.
 8. $2b, -2a$; ± 123 ; ± 275 . 10. ± 2 ; $\pm 2\frac{1}{2}$; $\pm 2\frac{1}{2}$.
 12. ± 5 ; $\pm \frac{2}{3}a$; $\pm \sqrt{a}$. 13. $\pm \sqrt{mn}$; $\frac{2}{3}, \frac{1}{3}$; ∞ .
 15. $12, 8$; $\pm \sqrt{\frac{a+c}{c-1}}$.
 16. $\pm \sqrt{ac} \div \sqrt{bd}$; $\pm \sqrt{ab}$; ± 1 .
 17. ± 1 ; $2a, 2b$. 18. $\pm \sqrt{m^2 + n^2}$; $\pm \sqrt{a^2 + b^2}$.
 19. $\pm \sqrt{a^2 + 2ab - 3b^2}$; $\pm \sqrt{a^2 + ab + b^2}$. 20. ± 1 .

EXERCISE LXXVII. (PAGE 241.)

1. $11\frac{1}{2}$. 2. ± 385 . 3. ± 13 .
 4. $\pm 77, \pm 91$. 5. $\pm \sqrt{10}$. 6. 78, 52, 39.
 7. 481, 259. 8. $\pm \sqrt{3}, 0$. 9. 2, 4, 8.
 10. $\pm \sqrt{6}$. 11. 15 or 21. 12. $a \pm \sqrt{a^2 - b}$.
 13. 40 ft. 14. 8, 12, and $-40, 60$. 15. 12 ft.
 16. 140, 120. 17. 20 per cent. 18. 960, 280.
 19. 285, 152. 20. 4 hrs.

EXERCISE LXXIX. (PAGE 250.)

1. $-7, 1; 6, 2; -7, -3.$
 2. $-2, -1; -m \pm \sqrt{(m^2 + n)}; \frac{1}{2}\{m \pm \sqrt{(m^2 + 4n)}\}.$
 3. $12, -2; -6, -20; 30, -4.$
 4. $23, -1; \frac{3}{2}, -\frac{1}{2}; \frac{1}{2}, -4.$ 5. $2\frac{2}{3}, 1; -\frac{1}{4} \pm \frac{1}{4}\sqrt{-543}.$
 6. $1\frac{15}{8}, 9; \frac{51 \pm \sqrt{1721}}{40}; -5 \pm \sqrt{2}.$ 7. $6\frac{1}{2}, -4\frac{97}{108}; 3\frac{1}{4}, -2\frac{1}{4}.$
 8. $4, \frac{4}{3}; 2, 2\frac{2}{3}; 28, -3.$ 9. $\frac{3}{2}\frac{2}{7}; -1\frac{1}{6}, 4.$
 10. $-\frac{2}{15}, 2; -23, 1.$ 11. $\frac{1}{2}(-5 \pm \sqrt{65}); -2a, a.$
 12. $a(a-b), b(a+b); -1, m \div (m-n).$
 13. $56\frac{1}{2}, 12\frac{3}{4}; (m+n)^2, -(m-n)^2.$ 14. $a+b, a; a+b, 0.$
 15. $a \pm b; -2a, 2b; 3, a^2 \div (6-a).$

EXERCISE LXXX [a]. (PAGE 257.)

1. $4, 3; -\frac{3}{2}, 1; -\frac{7}{4}, 4; -2\frac{1}{2}, 2\frac{1}{2}.$
 2. $-\frac{7}{4}, 4; -5, -\frac{3}{2}; 3\frac{1}{2}, 5.$ 3. $-\frac{3}{2}, 6; -\frac{5}{2}, 7; \frac{1}{3}, -2.$
 4. $30, -4; 6 \pm \sqrt{-64}; 2\frac{2}{3}, -\frac{5}{3}.$ 5. $a, b; b, a.$
 6. $c \div (a-b), d \div (a-b); 3, -5.$
 7. $2, -3\frac{1}{2}; a+b, \frac{ab}{a+b}.$

[b.] (PAGE 258.)

NOTE.—*Imaginary roots generally omitted.*

1. $\pm \sqrt{2}; \pm 1, \pm 2; \pm 2, \pm 3.$
 2. $\pm \sqrt{13}; \pm 2, \pm \sqrt{10}; -\frac{3}{7}, -\frac{9}{7}.$
 3. $2, -\sqrt[3]{11}; 2, 2\frac{1}{2}.$ 4. $3, 2; \pm 11.$ 5. $-2, 4; 2; 1.$
 6. $\pm \sqrt{\{+ \frac{1}{2}(-3 \pm \sqrt{201})\}}; x^2 + 5 = 2, \text{ or } -1.$
 7. $(\pm \sqrt{2}) \div 4; -2a, a \sqrt[3]{2}; x^2 = \frac{1}{2}\{a^2 \pm \sqrt{(a^4 - 4b^4)}\}.$
 8. $-1, 2; \pm \frac{3}{2}, \pm \frac{1}{4}\sqrt{11}; 1, -2.$
 9. $\frac{1}{2}\{-7 \pm \sqrt{41}\}, \frac{1}{2}\{-7 \pm \sqrt{21}\}; -4, 3, 3 \pm 21.$
 10. $\frac{1}{2}\{1 \pm \sqrt{(4a-3)}\}; 1, 2, 4.$ 11. $-1; -1, -2, 2\frac{1}{2}; 1.$
 12. $x^3 - 8 + x^2 - 4 = 0, x = 2; 3; 0, -1, -1.$
 13. $1, \frac{1}{2}\{p-1 \pm \sqrt{(p^2-2p-3)}\}; 1, \frac{1}{2}\{2p-1 \pm \sqrt{(4p^2-4p-3)}\};$
 $1, x^2 + x + 1 = 0.$

14. $-1, \pm 2; a, -a, -b; 0, \frac{1}{2}, -\frac{1}{2}$.
 15. $1, 1, \frac{1}{2}(-3 \pm \sqrt{5}); 2, \frac{1}{2}, -2 \pm \sqrt{3}$;
 equation is $\left(x + \frac{1}{x}\right)^2 + 1\frac{1}{2}\left(x + \frac{1}{x}\right) - 10 = 0$, or
 $2y^2 + 3y - 20 = 0$ (if $y = x + \frac{1}{x}$), $(2y - 5)(y + 4) = 0$, etc.

[c.]

1. $7, -2$. 2. $\frac{2}{3}, 3$. 3. $2, -22$.
 4. $a + 2c, -\frac{a(a+b)}{a+2c}$. 5. $\pm \sqrt{6}, \pm \sqrt{11}$. 6. $0, -\frac{2}{3}$.
 7. $3, -1$. 8. $5, -4\frac{2}{3}$. 9. $0, \pm 6$. 10. $-2\frac{1}{2}$.
 11. $\frac{\sqrt{\{(a-b)(c+d)\}} \pm \sqrt{\{(a+b)(c-d)\}}}{\sqrt{\{(a-b)(c+d)\}} \mp \sqrt{\{(a+b)(c-d)\}}}$. Use Art. 133.
 12. $-\frac{2}{3}$. 13. $\pm 8, \pm 1$ (put y for $x^2 - 8$ in the denominators).
 14. Complete the divisions, transpose and divide by 3, and

$$\frac{x^2 - \frac{1}{2} - 9x - 3}{x + 3} = x - 9\frac{1}{2}, 2\frac{1}{2}x = 25, x = 10.$$
 15. $4, 0$.
 16. $(4a - 5b) \div 6ab, (a - 2b) \div 3ab$.
 17. $\pm \frac{3}{2}\sqrt{-1}, \pm \frac{5}{2}\sqrt{-1}$.
 18. Use first quantity for the *unknown*; $7, 1, 4, 4$. 19. $1, 3$.
 20. $\pm 3\sqrt{\pm 1}, \pm 2\sqrt{\pm 1}$. 21. 2 . 22. $c, c - \frac{1}{2}(a + b)$.
 23. Complete the divisions, right member cancels, and

$$\frac{1}{x-9} + \frac{1}{x-25} - \frac{2}{x-49} = 0, \text{ or}$$

$$\frac{1}{x-9} - \frac{1}{x-49} = \frac{1}{x-49} - \frac{1}{x-25}; x = 19.$$

 24. Separate the factor $\frac{1}{x+a+b}$; $x = \frac{a^2 + b^2}{a+b}$.
 25. -1 , and $n(x^2 - x + 1) + 1 = 0$.
 26. $\frac{1}{2}\{-1 \pm \sqrt{45}\}$ ($+x$ should be $-x$ in given equation);
 put equation in the form $(x^2 + x)^2 - (x^2 + x) = 132$, etc.
 27. $\pm 2a, \pm 2a\sqrt{-1}$. 28. $2, 2\frac{1}{2}$. 29. $4, -\frac{5}{2}$. 30. $8, -\frac{7}{2}$.

[d.] (PAGE 260.)

1. $3, \frac{1}{2}, \pm \sqrt{-1}$ (see second equation in [b] 15).
 2. Equation is $(x+1)(x^2 - x + 1 \pm ax)$;
 $x = -1, \frac{1}{2}\{1 - a \pm \sqrt{(a^2 - 2a - 3)}\}$.

3. $-1, -\frac{1}{2}, -2$. 4. $2, -1, -4$. 5. $-6, 4$.
6. $-2\frac{1}{2}, \infty$. 7. $3, \infty$. 8. $3, \frac{1}{3}, \frac{1}{2}(1 \pm \sqrt{-3})$.
9. Take together pairs of like numerators and equation reduces to $x^4 - 5x^2 + 6 = 0 = (x^2 - 3)(x^2 - 2)$.
10. $-4, -2, 3 \pm \sqrt{7}$; observe that one fraction is reciprocal of the other; put it equal y , then $y + \frac{1}{y} = 2\frac{1}{2}$, etc.
11. Use formula (4), p. 181, and factor by Ex. 7, p. 105;
 $x = -2 \pm \sqrt{3}$.
12. $-\frac{1}{2} \pm \sqrt{\frac{5}{2}}$. 13. $2 \pm \sqrt{2}$, and $x^2 - 6x + 6 = 0$.
14. $1 - p$, equation reduces to $(x + p - 1)\left(x^2 + x + \frac{1}{p-1}\right) = 0$.
15. $7, 2$; if $x^2 - 9x + 18 = y$, equation is $y^2 + 2y = 24$, etc.
16. Divide by left member and clear, $\therefore 63(1+x) = 62(1-x)$, etc.
17. $a\left(1 \pm 2\sqrt{\frac{b}{ac}}\right)$, or 0; complete the division in left member, square, and 1 cancels.
18. $\frac{1}{2}$. 19. Sign before last fraction in left member should be $-$; equation reduces to $7x^2 - 34x + 9 = 0$.
20. Write $-$ for $+$ before 50 in denominator; combine first pair fractions and second pair; factor denominators and cancel; then taking $x^2 - 5x = y$, $y^2 + 18y - 24 = 0$, etc.
21. $\frac{1}{2}$. 22. Combine first and last fractions, second and last but one, etc., and $2x + 7$ is found to be a factor; then put $x^2 + 7x = y$, and resulting equation is $y^2 + 18y + 90 = 0$; $x = -3\frac{1}{2}$, etc.
23. Combine in pairs like numerators, then as in last example; $2\frac{1}{2}$,
 $2\frac{1}{2} \pm \frac{1}{2}\sqrt{[5a + 13b + 17c \pm \sqrt{\{(a - 3b - 2c)^2 + 12ab\}}]$
 $\div (a + b + c)]$.
24. Combine first and second, etc.; $-\frac{1}{2}(a + b)$, ∞ ,
 $-\frac{1}{2}(a + b) \pm \frac{1}{2}\sqrt{\{\frac{1}{2}(a + b - 2c)^2 + \frac{1}{2}(a - b)^2\}}$.
25. See Ex. 8; $n, \frac{1}{n}, -n \pm \sqrt{(n^2 - 1)}$.

EXERCISE LXXXI [a]. (PAGE 264.)

1. 11 or - 24.
2. 26 and 19.
3. $\frac{4}{25}$ or - $6\frac{1}{2}$.
4. $\frac{1}{2}(\sqrt{5} - 1)a$, $\frac{1}{2}(3 - \sqrt{5})a$.
5. $16\frac{1}{2}$.
6. 50 coffee, 60 raisins.
7. \$240.
8. 100.
9. 12.
10. 11 vases.
11. A, 11 miles; B, 10.
12. 3, 4, and 5.
13. 25 cts.
14. 4d. a dozen.
15. 3 and 18.
16. 7.
17. A, 72 miles; B, 54 miles.
18. \$90 or \$10.
19. $\frac{21}{5}$.
20. A, \$1800; B, \$1600.
21. \$3.
22. $\frac{1}{2}\{\sqrt{(2h^2 - d^2)} + d\}$, $\frac{1}{2}\{\sqrt{(2h^2 - d^2)} - d\}$.
23. 2.414 inches.

[b.] (PAGE 266.)

1. 3 hours and 5 hours.
2. 36 and 30.
3. 63.
4. $-\frac{1}{4}(a+b) \pm \sqrt{\left(\frac{ab}{4m} + \frac{1}{16}(a+b)^2\right)}$.
5. 901.
6. 4200; read 780 in question.
7. 14 acres at \$75.
8. 10 seconds.
9. $5\frac{1}{2}$ miles.
10. 5 miles an hour.
11. 15 miles.
12. If x be cost and s selling price, then $x = s + \frac{x^2}{100}$; on solving it is seen that $4s$ cannot be greater than 100; see Art. 175 (i).
13. \$333\frac{1}{3}, \$666\frac{2}{3}
14. 72, 12, 8; Let x^2 = number remaining in smaller bag after handful is taken; then x^6 is left in larger bag, and x^3 = number in handful, and x^4 is number in larger after second lot is taken out; then $x^4 + x^2 = \frac{5}{3}(x^2 + x^6)$, and $x = 2$, etc.
15. If x represents per cent, then $620 = 82x + (3790 + 82x)\frac{x}{100}$.
 $x = 5$.
16. Let $2x$ = distance, then $\frac{x}{x-6} + \frac{x}{x-4} = \frac{6}{7} \cdot \frac{2x}{x-6}$; $x = 22$.

17. Let x = rate backwards, $4x$ = rate forwards, then .

$$\frac{\frac{3}{4}}{4x} + \frac{\frac{1}{4}}{x}, \text{ i. e. } \frac{7}{16}x = \frac{\frac{3}{4}}{4x+2} + \frac{\frac{1}{4}}{x-\frac{1}{2}}; \text{ 1 mile an hour.}$$

18. 90. 19. 4900; x^2 being number of lines, equation is

$$\left\{\frac{4}{3}(x-10)\right\}^2 = 2\left\{x^2 - \frac{4}{3}(x-10)\right\}, \text{ or } 601x^2 - 20x \cdot 2254 \\ = -100 \cdot 49 \cdot 48; \quad x = 70.$$

20. A, half-past 4 o'clock; B, 5 o'clock.

EXERCISE LXXXII [a]. (PAGE 272.)

1. $x = \pm b, y = a \pm b.$
2. $x = \pm b, y = a \mp b.$
3. $x = 3, -\frac{2}{3}, y = 1, -1\frac{2}{3}.$
4. $x = 3, -2\frac{1}{2}, y = 3, -8.$
5. $x = 4, -10\frac{1}{4}, y = 5, -12\frac{3}{4}.$
6. $x = \pm 7, y = \pm 2.$
7. $x = \pm 20, y = \pm 16.$
8. $x = \pm 15, y = \pm 3.$
9. $x = 30, 10, y = 10, 30.$
10. $x = 1, -5, y = -1, -7.$
11. $x = 3, -3\frac{1}{4}, y = 4, -3\frac{3}{4}.$
12. $x = 2, -\frac{1}{3}, y = 3, \frac{2}{3}.$
13. $x = 1, \frac{1}{4}, y = 2, 2\frac{1}{4}.$
14. $x = 10, 115, y = 6, -69.$
15. $x = 7, -4\frac{3}{4}, y = 3, -2\frac{7}{4}.$
16. $x = 1, -\frac{5}{8}, y = -4, 8\frac{1}{2}.$
17. $x = \pm 7, \pm 5, y = \pm 11, \pm 9.$
18. $x = 2, 3, y = 5, 4.$
19. $x = 2, 5, y = 6, 3.$
20. $x = 5, \frac{3}{4}, y = 3, -1\frac{1}{4}.$
21. $x = 7, 1, y = 3, 9.$
22. $x = \pm \sqrt{13}, y = \pm \sqrt{13}.$
23. $x = 3, y = 1.$
24. $x = 2, \frac{5}{8}, y = -7, -\frac{1}{8}.$

[b.]

1. $x = \pm 5, y = \pm 1.$
2. $x = \pm 11, y = \pm 2.$
3. $x = 0, \pm 2, y = \pm \sqrt{\frac{11}{3}}, \pm 1.$
4. $x = 0, \pm 3, y = \pm 3, \pm 9.$
5. $x = \pm 3, \pm \frac{5}{3}, y = \pm 5, \pm 1\frac{2}{3}.$
6. $x = 0, \pm 1, y = \sqrt{\frac{1}{2}}, \pm 1.$
7. $x = \pm 2\frac{1}{3}, y = \pm \frac{1}{3}.$
8. $x = \pm 3\sqrt{\frac{3}{7}}, y = \pm \frac{1}{3}\sqrt{\frac{3}{7}}.$
9. $x = \pm 2\frac{1}{3}, \pm 1, y = \pm 1, \mp 3.$
10. $x = \pm 7, \pm 3, y = \pm 2, \pm 6.$
11. $x = \pm 1, \pm 1\frac{1}{2}, y = \pm 5, \mp 1\frac{2}{3}.$
12. $x = \pm a \div \sqrt{(a+b)}, y = \pm b \div \sqrt{(a+b)}.$
13. $x = \pm 6, y = \pm 2.$
14. $x = \pm 3, \pm 8, y = \pm 5.$
15. $x = \pm 2, \pm \sqrt{\frac{2}{3}}, y = \pm \frac{1}{2}, \mp 2\sqrt{\frac{2}{3}}.$
16. $x = \pm 4, \pm 3\sqrt{3}, y = \pm 5, \pm \sqrt{3}.$
17. $x = 7, 4, y = 4, 7.$
18. $x = 7, -5, y = 5, -7.$

19. $x = \pm 5, \pm 3, y = \pm 2, 7.$ 20. $x = \pm 5, \pm 4, y = \pm 3.$
 21. $x = 4, y = 3.$ 22. $x = 14, 19, y = 19, 14.$
 23. $x = 5, 4, y = 4, 5.$ 24. $x = 4, 2, y = 2, 4.$
 25. $x = 4, y = 3.$ 26. $x = 3, 2, y = 2, 3.$
 27. $x = 13, 9, y = 9, 13.$ 28. $x = 7, 4, y = 4, 7.$
 29. $x = 3, 1, y = 1, 3.$ 30. $x = 3, 2, y = 2, 3.$

EXERCISE LXXXIII [a]. (PAGE 277.)

1. $x = 2, 1, y = -1, -2.$ 2. $x = 4, -3, y = -3, -10.$
 3. $x = \pm \frac{2}{3}, \mp \frac{1}{3}, y = \frac{4}{3}, 0.$ 4. $x = \pm 1, y = \pm 7.$
 5. $x = \pm 5, y = \pm 2.$ 6. $x = \pm 1, y = \pm \frac{1}{2}.$
 7. $x = \frac{1}{2}(a \pm 2b), y = \frac{1}{2}(a \mp 2b).$
 8. $x = \pm (a + b), y = \pm (a - b).$
 9. $x = \pm \frac{a^2 + b^2}{a - b}, y = \pm \frac{2ab}{a - b}.$
 10. $x = \frac{1}{2}, y = \frac{1}{2}.$ 11. $x = 6\frac{1}{2}, y = 1\frac{1}{2}.$
 12. $x = 0, a, y = a, 0.$ 13. $x = \pm 9, \pm 5, y = \pm 5, \pm 9.$
 14. $x = \pm 7, \pm 3, y = \pm 3, \pm 7.$
 15. $x = \pm 4, \pm 3, y = \pm 3, \pm 4.$
 16. $x = 2, -1, y = 1, -2.$ 17. $x = 11, y = 9.$
 18. $x = -2, -3, 3 \pm \frac{1}{2}\sqrt{56}, y = -3, -2, 3 \mp \frac{1}{2}\sqrt{56}.$
 19. $x = 5, -1, \frac{1}{2}(\pm \sqrt{41} + 5), y = 1, -5, \frac{1}{2}(\pm \sqrt{41} - 5).$
 20. Treat $x + y$ as the unknown; $x = \frac{1}{2}\{a \pm \sqrt{(a^2 - 48)}\},$
 $y = \frac{1}{2}\{a \mp \sqrt{(a^2 - 48)}\},$ where $a = \frac{1}{2}(-3 \pm \sqrt{853}).$
 21. $x = \frac{1}{2}(9 \pm \sqrt{-47}), y = \frac{1}{2}(9 \mp \sqrt{-47}).$
 22. $x = 5, -2, -\frac{1}{2}(1 \pm \sqrt{41}), y = 2, -5.$
 23. $x = 17, -6, \pm \sqrt{(118) - 4}, y = 3, -8\frac{1}{2}, 2 \pm \frac{1}{2}\sqrt{118}.$
 24. $x = 5, 6, 7, 8, y = 8, 7, 6, 5.$
 25. $x = \pm 13\sqrt{\frac{45}{218}}, y = \pm 7\sqrt{\frac{45}{218}}.$ 26. $x = 4, 2, y = 2, 4.$
 27. $x = 1, 10, y = 10, 1.$ 28. $x = 3, 2, y = 2, 3.$
 29. $x = 8, 4, y = 4, 8.$ 30. $x = 1, 1\frac{1}{17}, y = 2, -1\frac{1}{17}.$
 31. $(a'e - ac')^2 + (ab' - a'b)(b'e - bc') = 0.$

EXERCISE LXXXIV. (PAGE 281.)

1. $x = 8, 2, y = 4, z = 2, 8.$ 2. $x = \pm 4, \pm 9, y = \pm 6,$ etc.
3. $x = 7, y = 6, z = 5.$ 4. $x = 1, y = 2, z = 3.$
5. $x = \frac{4}{3}, y = \frac{4}{3}, z = 4.$
6. $x = \pm \frac{4}{3} \sqrt{3}, y = \pm \sqrt{3}, z = \pm 2 \sqrt{3}.$
7. $x = 4, -7, y = 3, -8, z = 6, 28, -2z$ in text.
8. $x = 1, 9, y = \pm 4, z = 2, -6.$
9. $x = 2, -14, y = 3, -15, z = 4, -16.$
10. $x = 1, y = -2, z = 4.$ 11. $x = 4, y = -5, z = 7.$
12. $x = 2, 7, y = 3, z = 7, 2.$ 13. $x = y = z = \pm 1 \div \sqrt{2}.$
14. $x = 2abc \div (ac + bc - ab), y, z$ symmetrical.
15. $x = \pm a^2 \div \sqrt{(a^2 + b^2 + c^2)}, y, z$ by symmetry.
16. $x = \pm \sqrt{\{(a + b - c)(a + c - b) \div 2(b + c - a)\}},$
 y, z by symmetry.
17. $x = abc \div (ab + ac - bc), y, z$ by symmetry.
18. $x = \sqrt{\{(c + a - b)(a + b - c) \div (b + c - a)\}}.$
19. Add n^2 to each equation and factor;
 $x = -n \pm (a + n)(c + n) \div (b + n), y, z$ by symmetry.
20. $x = \pm a \sqrt{\{(b + c - a) \div [(a + b - c)(a - b + c)]\}},$
 y, z by symmetry.
21. $x = (bc + ca + ab) \div a, 0; y, z$ by symmetry.
22. $x = 1, 2, 4, y = 2, 4, 1, z = 4, 1, 2.$
23. $x = \pm (-m + n + p) \div \sqrt{2(m + n + p)}; y, z$ by symmetry.
24. $x = \pm (-bc + ca + ab) \div \sqrt{(2abc)}; y, z$ by symmetry.
25. $x = 0,$ or $\pm 1 \div (c - a); y, z$ by symmetry.
26. $x = \sqrt[3]{(b^2c^2 \div a)}; y, z$ by symmetry.

EXERCISE LXXXV [a]. (PAGE 285.)

1. $\frac{1}{2}(1 + \sqrt{5}), \frac{1}{2}(3 + \sqrt{5}).$ 2. $\frac{1}{2} \pm \frac{1}{2} \sqrt{2193}, -\frac{1}{2} \pm \frac{1}{2} \sqrt{2193}.$
3. 36, 16, or -36, -16. 4. $\pm \sqrt{(pq)}, \pm \sqrt{(p \div q)}.$
5. 20. 6. $\pm \frac{1}{2}(p + q) \sqrt{(a \div pq)}, \pm \frac{1}{2}(p - q) \sqrt{(a \div pq)}.$
7. 7, 21, 35. 8. 343, 64.
9. $\frac{4}{3}, \frac{-1\frac{1}{2}}{\frac{1}{3}}.$ 10. 36.
11. 34, 17, 51, or -204, 612, -306. 12. 36.

13. $\sqrt[3]{119} \pm \frac{1}{2} \sqrt{6} \div \sqrt{119}$. Add and subtract the equations.
 14. 8 ft., 10 ft. 15. 88 yds., 55 yds.
 16. 63 ft., 45 ft. 17. 20 m., 30 m.
 18. 6%, 7%. 19. 102 from length, 114 to width.
 20. 18, 9.6. 21. 100 at \$75 each.
 22. 13, 10. 23. A 40 at \$1.20, B 30 at \$1.60.
 24. 3, 5, 10. 25. 3, 4, 5.

[b.]

1. Edges (x, y, z) are 1, 2, 4;
 $x + y + z = 7$, $x^2 + y^2 + z^2 = 21$, $x^3 + y^3 + z^3 = 73$.
 Cube first equation by formula H (3), p. 85, and substitute
 from third, second, and square of first.
 2. 864. 3. 2, 5, 8. See Ex. LXXXV, 16. Add first two
 equations and subtract third, then symmetry.
 4. 76; the *one digit* remainder is of course 9.
 5. \$2145, $2\frac{1}{2}$ years. 6. 4, 7, 10.
 7. 290 yds. 8. 48, 10.
 9. 8, 9, 10; see Ex. 4, p. 284.
 10. 342; in last line of problem read 29.
 11. 12, 4, 3. 12. 3, 6, 9. See LXXXIV, 19.
 13. $\frac{1}{2} \{b \pm \sqrt{(c-a^2)}\} : \frac{1}{2} \{a \pm \sqrt{(c-b^2)}\} :: \frac{1}{2} \{a \mp \sqrt{(c-b^2)}\}$
 $: \frac{1}{2} \{b \mp \sqrt{(c-a^2)}\}$.
 14. $x = \sqrt{\{(1+a)(1+b) \div (1+c)\}}$, y and z by symmetry.
 See 4, p. 284.
 15. 28 workmen, each 45 lbs., or 36 workmen and each 77 lbs.;
 x = number of workmen, y lbs. carried each load, z number
 loads in one hour; then $8xyz$ is whole weight moved, and
 $7(x+8)(y-5)z = 8xyz = 9(x-8)(y+11)z$.
 16. $\frac{b - \sqrt{(b^2 - 4p)}}{a - \sqrt{(a^2 - 4p)}} = \frac{a + \sqrt{(a^2 - 4p)}}{b + \sqrt{(b^2 - 4p)}}$, where p = product of ex-
 tremes (or means), and $= (a^3 + b^3 - c) \div 3(a+b)$.
 17. Of the first, 135, 62; of the second, 182, 57, (yds.). 18. 126.

EXERCISE LXXXVI [a]. (PAGE 296.)

1. $x^2 - 2x - 2 = 0$.
2. $\frac{3}{4}$.
3. $32x^2 - 1412x - 23205 = 0$.
4. $x^2 - a^2b = 0$.
5. $p^2 - q$; $p(p^2 - 3q)$; $(p^2 - 2q) \div q^2$; see Art. 178.
6. $x^2 - (4a - 6b)x + 9a^2 - 10ab + 8b^2 = 0$.
7. 3.14159.
8. Positive for *all* values of x , expression $= (x - 2\frac{1}{2})^2$.
9. 78949.
10. $p - 2q + 3r$.
11. See Ex. 1, p. 294.
16. Assume $x'' = Ax + Bq$, then, since α, β are values of x ,
 $\alpha'' = A\alpha + Bq$, and $\beta'' = A\beta + Bq$, whence A and B.

[b.]

2. $b^2 - ac = 0$.
4. 1. $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$;
 2. $q^2x^2 - (p^2 - 2q)x + 1 = 0$;
 3. $x^2 - (p^2 - q)x + q(p^2 - 2q) = 0$;
 4. $x^2 + \{p - \sqrt{(p^2 - 4q)}\}x - p\sqrt{(p^2 - 4q)} = 0$.
8. $cc' - aa' = 0$, $b'c + a'b = 0$.

[c.]

4. 579 and 135 are the roots of the first equation, 579 and -135 those of the second.
12. $4ab^2 + a'c - aa'c' = 0$; let roots of first equation be α, β , of second $\alpha + m, \beta + m$; form equations from relations of roots and coefficients and eliminate m .
13. (Right side of first equation should be 1.) Substitute for y in second equation, and apply condition of *equal roots* to resulting equation in x .

EXERCISE LXXXVII [a]. (PAGE 300.)

1. Min. 4.
2. Min. $-\frac{1}{4}$.
3. Max. $\frac{1}{4}$.
4. Min. $\frac{1}{18}$.
5. Min. $\frac{1}{80}$.
6. Min. $-\frac{1697}{14}$.
7. Min. $\frac{1}{4}$.
8. Min. 2.
9. Min. $-\frac{1}{2}$, Max. $\frac{1}{2}$.
10. Max. 36 area, *i. e.* line is *bisected*.

[b.]

2. 81. 3. Min. $\frac{1}{2}a^2$, *i. e.* line is bisected.
 4. $\frac{1}{2}a\sqrt{2}$, the sides are equal. 6. $(a+b)^2 \div 4ab$.
 7. All numbers between $\frac{1}{2}$ and 3. 8. $\frac{1}{2}\sqrt{1+\frac{1}{2}}$.
 11. $(b^2 - 4ac) + a^2 = (n^2 - 4mr) + m^2$. 12. $p = 6$, or $\frac{2}{3}$.

EXERCISE LXXXVIII [a]. (PAGE 305.)

1. 1. 2. $24b^3$. 3. 5. 4. 1.
 5. $\sqrt[3]{a^2}$; $2\sqrt[4]{a^3}$; $5a^3\sqrt{b}$; $7\sqrt{ab^3}$;
 $6\sqrt{a}\sqrt[4]{b}$; $\sqrt{a}\sqrt[3]{b}\sqrt[4]{c}$; $\sqrt[4]{a^3}\sqrt[7]{b^6}\sqrt[3]{c^{10}}$.
 6. $a^{\frac{1}{2}}$; $a^{\frac{2}{3}}b^{\frac{1}{3}}$; $a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{6}}$; $a^{\frac{2}{3}}b^{-\frac{1}{3}}c^{\frac{1}{6}}$; $a^{\frac{1}{2}}b^{\frac{1}{3}}c^{-\frac{1}{6}}d^{\frac{1}{6}}$.
 7. 1. a^2b^{-3} ; $a^{-1}b^{-1}c$; $a^{\frac{1}{2}}b^{-2}$; $7a^{\frac{1}{2}}b^{\frac{1}{3}}$; $a^{-2}b^{-2}$; a^2b^3 ;
 $5a^3b^7c^{-5}$; $6a^{\frac{5}{4}}b^{\frac{1}{2}}c^{-\frac{7}{8}}$.
 2. $\frac{1}{a^{-2}b^3}$; $\frac{1}{abc^{-1}}$; $\frac{1}{a^{-\frac{1}{2}}b^3}$; $\frac{7}{a^{-\frac{1}{2}}b^{-\frac{1}{4}}}$; $\frac{1}{a^2b^3}$; $\frac{1}{a^{-2}b^{-2}}$;
 $\frac{5}{a^{-5}b^{-7}c^5}$; $\frac{6}{a^{-\frac{5}{4}}b^{-\frac{1}{2}}c^{\frac{7}{8}}}$.

[b.]

1. $(a^2 - b^2)^n$; $(x+y)^{n-1}$; $(x-y)^n$. 2. $\frac{2}{3}a^{\frac{4}{3}}$; 1.
 3. $x^{\frac{4}{3}} - 4x^{\frac{1}{3}}$; $a^{\frac{1}{3}}b - a^{\frac{5}{3}}b^3 + 3a^2b^{\frac{1}{3}}$;
 $ab^{\frac{2}{3}} + a^{\frac{2}{3}} - a^2b^{\frac{1}{3}} - a^{\frac{5}{3}}b^{\frac{1}{3}}$.
 4. $a^{\frac{1}{2}}b^{\frac{1}{3}} - 5ab^{\frac{2}{3}} + a^{\frac{4}{3}}b^{\frac{1}{3}} - a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{6}}$; $a^{-\frac{1}{2}}$; $3y^{-\frac{2}{3}}$; $x^{\frac{4}{3}}y^{\frac{1}{3}}$.
 5. $\frac{c}{a^2b^3} - \frac{a}{b^3} + \frac{b}{a^4c^3} - \frac{1}{ab^2c^3}$; $\frac{c}{a^{\frac{1}{2}}b^{\frac{1}{3}}} + \frac{b^{\frac{2}{3}}c^{\frac{1}{3}}}{a^{\frac{2}{3}}} + \frac{1}{b^{\frac{2}{3}}}$.
 6. $\frac{c^3}{a^2b^3} + 2abc - \frac{3a^3}{b^2c^3} + a^2b^2c^3$; $\frac{x^2c^4}{a^3y^3} - \frac{x^{a+b}}{y^{a+b}} + \frac{y^4}{5x}$.
 7. $\sqrt[5]{x^3} + \sqrt[3]{y^2} - \sqrt[5]{z^3}$; $\sqrt[3]{x^{-2}} \cdot \sqrt[4]{y^3}$; $21\sqrt{a^{-6}}$;
 $\frac{3\sqrt[3]{b^2}}{4\sqrt[4]{a^5}}$.

8. $\frac{\sqrt[3]{c}}{\sqrt[3]{a^2b^2}} + \frac{\sqrt{a^3}}{\sqrt{b^2}} - \frac{\sqrt[3]{a^2}}{\sqrt[3]{b^2}} + \frac{\sqrt{b}}{\sqrt{a}};$
 $\sqrt[3]{x} \cdot \sqrt{a^3}; \sqrt[3]{a^2}; \sqrt[3]{a^{11}}.$
9. $8; \frac{1}{8}; \frac{1}{25}; \frac{1}{16}; \frac{1}{11}; \frac{1}{5}.$ 10. $1024a; a + 32.$
11. $\frac{625a^6}{384b^2}; a^{6x-9y} \cdot b^{10x-12y} \cdot c^{14x-16y}.$ 12. $(7x-6y)^{\frac{1}{3}}; (5a-7b)^2.$
13. $244140625; 2.$ 14. $0.$

[c.] (PAGE 306.)

1. $\frac{b^{\frac{2}{3}}}{a^{\frac{2}{3}}}; a^{\frac{2}{3}}b^{\frac{4}{3}}; \frac{81a^6}{16a^6}; x^{\frac{1}{12}}.$ 2. $x^{2n-1}; a^{\frac{5}{4}}b^{\frac{3}{8}}; \frac{1}{(x-y)^2}.$
3. $a^{14}b^{-23}; a^{\frac{1}{6}}b^{-\frac{1}{6}}; \left(\frac{x}{y}\right)^7.$ 4. $a^2b^{\frac{m^2+n^2}{mn}}; b^{\frac{m^2-n^2}{mn}}; a^{11mnp}; b^{m^2-mn}.$
5. $a^{-\frac{1}{4}}; a^{12b^4}; x^{-4}y^4; a^{\frac{m^2+n^2}{mn}} \cdot b^{\frac{m+n}{n}}.$ 6. $a^3b^4c^2; a^2b^3c.$
7. $\left(\frac{a^{\frac{5}{3}}b^2}{c^{\frac{2}{3}}}\right); -\frac{x^6}{y^6}; a^{\frac{4}{3}}b^{\frac{3}{8}}b^{\frac{7}{8}}.$

EXERCISE LXXXIX [a]. (PAGE 309.)

1. $x^{a+b+c}; x^{a^2+a+1}; x^{m^2n^2}; a^{\frac{m^2}{m+1}}.$
2. $2^{n^2}; \frac{1}{b}; (a^{\frac{1}{n}} \cdot b^{-\frac{1}{m}})^{pq}.$
3. $x^{\frac{2}{3}} + x^4 + x^{\frac{7}{3}} + 2x^3 + x^{\frac{5}{3}} + x^2 + x^{\frac{3}{2}} + x + 1; x + y.$
4. $x^{\frac{3}{2}} - y^{\frac{3}{2}}; x^2 + y^2.$
5. $x^4 + 2x^2y^2 + y^4 - xy; x^2 - 2xy + y^2.$ 6. $4a^2 - b^3.$
7. $2x^{2a} - 49 - 9x^a + 6x^{-2a} + 11x^{-a}; 4x^{\frac{2m}{n}} - 9y^{\frac{2p}{q}}.$
8. $a^3 + a^{-3} - 2 - a^{\frac{4}{3}} - a^{-\frac{2}{3}} + 2a^{\frac{1}{3}}.$
9. $x^3 + 4x^2y^{\frac{2}{3}} - 4x^2y - 16xy^{\frac{4}{3}} + 16xy^{\frac{5}{3}} - 64y^2.$
10. $3 + 2x^{-\frac{n}{2}} + 2x^{\frac{n}{2}} + x^{-n} + x^n; a^{\frac{1}{3}}x^{\frac{1}{4}} - b^4.$
11. $x^{\frac{1}{2}} - y^{\frac{1}{2}}; x + y + x^{\frac{1}{2}}y^{\frac{3}{2}} + x^{\frac{3}{2}}y^{\frac{1}{2}}.$
12. $x^{\frac{4}{3}} - x^{\frac{2}{3}}y^{\frac{1}{3}} + x^{\frac{2}{3}}y^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}} + y^{\frac{4}{3}}; 8a^{-2} + 7a^{-1} + 6.$
13. $5b^{\frac{1}{2}} + 4b^{\frac{1}{2}} + 3b^{-\frac{1}{2}} + 2b^{-\frac{1}{2}}.$

14. $x^{-\frac{5}{2}} + x^{-\frac{1}{2}}y^{-\frac{1}{2}} + x^{-1}y^{-\frac{3}{2}} + x^{-\frac{3}{2}}y^{-1} + x^{-\frac{1}{2}}y^{-\frac{5}{2}} + y^{\frac{5}{2}};$
 $a^{-\frac{5}{2}} - 2a^{-\frac{3}{2}}b^{\frac{1}{2}} + 4a^{-\frac{1}{2}}b^{\frac{3}{2}} - 8a^{-1}b + 16a^{-\frac{3}{2}}b^{\frac{5}{2}} - 32b^{\frac{5}{2}}.$
15. $x^{\frac{1}{2}} - 4x + 10x^{\frac{3}{2}} - 16x^{\frac{5}{2}} + 19 - 16x^{-\frac{1}{2}} + 10x^{-\frac{3}{2}} - 4x^{-1} + x^{-\frac{5}{2}}.$
16. $(a^{\frac{1}{2}} - x^{\frac{1}{2}}y^{\frac{1}{2}} + y^{\frac{3}{2}})(x^{\frac{3}{2}} + x^{\frac{1}{2}}y^{\frac{1}{2}} + y^{\frac{5}{2}});$
 $(x^{\frac{3}{2}} - 2x^{\frac{1}{2}}y^{-\frac{1}{2}} + 2y^{-\frac{3}{2}})(x^{\frac{3}{2}} + 2x^{\frac{1}{2}}y^{-\frac{1}{2}} + 2y^{-\frac{3}{2}}).$
17. $(x^{\frac{1}{2}} - 8)(x^{\frac{1}{2}} + 7); (3x^{\frac{3}{2}} - y^{\frac{1}{2}})(3x^{\frac{3}{2}} + 2y^{\frac{1}{2}}).$
18. $(x - 1)(x - x^{\frac{1}{2}} + 1); (3x^{\frac{1}{2}} - 2y^{\frac{1}{2}})(2x^{\frac{1}{2}} - 3y^{\frac{1}{2}}).$
19. $-a^{-1}(1 + b^{-1}), b^{-1}$ in denominator; $\left(\frac{a}{b}\right)^{\frac{1}{2}(m+1)};$

$$\frac{a^{\frac{2}{3}} + b^{\frac{2}{3}} + c^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{1}{3}} - b^{\frac{1}{3}}c^{\frac{1}{3}} - c^{\frac{1}{3}}a^{\frac{1}{3}}}{a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}}.$$
20. $ab^{-1} + 1 + a^{-1}b.$ 21. $a^{\frac{2}{3}} + 2a^{\frac{1}{3}}y^{\frac{1}{2}} + 3y^{\frac{2}{3}}.$

[b.] (PAGE 310.)

1. 0; expression = $(ab - a^{-1}b^{-1})(ab + a^{-1}b^{-1}) - ab(ab^{-1} + a^{-1}b) + a^{-1}b^{-1}(a^{-1}b + ab^{-1}) = (ab - a^{-1}b^{-1})(ab + a^{-1}b^{-1}) - (ab^{-1} + a^{-1}b) \times (ab - a^{-1}b^{-1}) = (ab - a^{-1}b^{-1})(ab + a^{-1}b^{-1} - ab^{-1} - a^{-1}b) = 0.$
2. See last question.
3. For last term read $a^{\frac{5}{2}}.$ Ans. $a^{\frac{1}{2}} - 2a^{\frac{3}{2}} + a^{\frac{5}{2}}.$
4. $(x^{47}y^{49})^{\frac{1}{2}}.$ 5. $2x(3 + x^2) \div (1 + x)^3.$
6. $a^{\frac{4}{3}} - 2ab^{\frac{1}{3}} + 3a^{\frac{2}{3}}b - 2a^{\frac{1}{3}}b^{\frac{2}{3}} + b^2;$ in divisor read $2a^{\frac{1}{3}}b^{\frac{1}{3}}.$
7. $x^4 + 2x^3 - 8x^2 - 6x - 1.$
8. $ax^3 - a^{\frac{1}{2}}x = a^{\frac{1}{2}}x(a^{\frac{1}{2}}x^2 - 1),$ and second factor of this is contained in the product of the other two quantities,
 $\therefore \text{L. C. M.} = a^{\frac{1}{2}}x(a^2x^6 - 1).$
9. $ab^{-1} + 1 - \frac{1}{2}a^{-1}b;$ in second term in text read b^3 for $b^{-2}.$
10. $\frac{1}{17};$ in first term read $a^{\frac{2}{3}}.$
11. $\frac{(x - a^{\frac{1}{2}})(x + a)}{x - a};$ in second term of numerator read $a.$
12. $\sqrt[12]{(x^5y^6)} \div \sqrt[5]{(a^2b^2)}.$

13. Expression $= a^{\frac{2}{3}} (a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{1}{3}} + b^{\frac{2}{3}} (a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{1}{3}} = \text{etc.}$

14. Numerator $= x^2 (x^{\frac{m-n}{n}} - x^{\frac{n-m}{n}}),$

Denominator $= \text{same factor} \times (x^{\frac{m}{n}} + x^{\frac{n}{n}}).$

15. $a^{6x} + a^{5x-y} + a^{4x-2y} + a^{3x-3y} + a^{2x-4y} + a^{x-5y} + a^{-6y}.$

16. $abcde$; divisor $= d^{10}$ (say) $= d^{17} \times d^2$, apply Law IV.

17. $x^{\frac{1}{2}} + (1-m) a^{\frac{1}{2}} x^{\frac{1}{2}} + a.$

18. Expression $= x^2 - y^2 - (y^2 - x^2) = xy(xy^{-1} - x^{-1}y)$
 $- x^{-1}y^{-1}(xy^{-1} - x^{-1}y), \text{ etc.}$

19. $a^{\frac{1}{2}}x + 2$; second remainder is $-5(ax^2 + 7a^{\frac{1}{2}}x + 10)$
 $= -5(a^{\frac{1}{2}}x + 2)(a^{\frac{1}{2}}x + 5), \text{ and H. C. F. is } a^{\frac{1}{2}}x + 2.$

20. $(ax^2 - 1)(ax^2 + 1)(a^3x^6 - 1) = a^5x^{10} - a^3x^6 - a^2x^4 + 1.$

21. $1 - \frac{1}{8}a^{-3}x^6.$

22. $a^{\frac{1}{3}}x(a^{\frac{1}{3}}x - 1) \div (2a^{\frac{1}{3}}x - 1$; read x^3 in first term of numerator;
 denominator $= (3a^{\frac{1}{3}}x + 1)(2a^{\frac{1}{3}}x - 1)$;
 $(1 - a^{\frac{2}{3}}) \div \{a(5a + a^{\frac{1}{3}})\}$;
 denominator $= a^{\frac{2}{3}}(a + a^{\frac{1}{3}})(5a + a^{\frac{1}{3}}).$

23. $x - y + z = - \left\{ \frac{1}{(b-c)(c-a)} + \frac{1}{(a-b)(b-c)} + \frac{1}{(a-b)(c-a)} \right\}$
 $= 0;$

Expression $= \frac{-(a-b)(b-c)(c-a)}{(a-b)(b-c)(a-c)} = 1.$

24. $(a+b) \frac{m^2 + 3n^2 + m - n}{2mn}.$

EXERCISE XC. (PAGE 313.)

1. $\sqrt[3]{x^2}$; $7\sqrt{(xy^3)}$; $5\sqrt[4]{(x^3y^7)}$; $6\sqrt[6]{(x^2y^4)}$; $\sqrt{(a^3b^4)}.$

2. $27^{\frac{1}{3}}$; $512^{\frac{1}{5}}$; $9^{-\frac{1}{2}}$; $(\frac{8}{9})^{\frac{1}{2}}$; $8^{\frac{1}{3}}.$

3. 1. $-\frac{2}{3}(b^3)^{\frac{1}{3}}$; $\frac{3}{4}(a^3b^3)^{\frac{1}{3}}$; $(4^{-9})^{\frac{1}{3}}$; $\{(\frac{3}{16})^2\}^{\frac{1}{2}}$; $(a^3b^{-3}c^{-9})^{\frac{1}{3}}.$

2. $-\frac{2}{3}(b^{-9})^{-\frac{1}{3}}$; $\frac{3}{4}(a^{-3}b^{-12})^{-\frac{1}{3}}$; $(4^{12})^{-\frac{1}{4}}$; $\{(\frac{1}{8})^4\}^{-\frac{1}{4}}$; $(a^{-4}b^4c^{12})^{-\frac{1}{4}}.$

4. $\sqrt{(36)}$; $\sqrt[3]{(250)}$; $\sqrt{\frac{1}{2}}$; $\sqrt{6}$; $\sqrt[3]{(90)}$; $\sqrt[3]{(16)}$; $\sqrt[4]{\frac{1}{2}}$;
 $\sqrt[3]{a^4}$; $\sqrt[4]{(a^2b^5)}.$

5. \sqrt{ab} ; $\sqrt{\frac{a}{b}}$; $\sqrt[3]{\frac{a^3}{b^3}}$; $\sqrt{(6a^3x)}$; $\sqrt[3]{\left(\frac{16a^3}{81b^3}\right)}$; $\sqrt[3]{\left(\frac{16a}{3}\right)}$;
 $\sqrt{(a^2 - b^2)}$.
6. $\sqrt[3]{(a^3b)}$; $\sqrt[3]{(a^{n+1})}$; $\sqrt[3]{\{(a^2 - x^2)^n(a + x)\}}$;
 $\sqrt{\left(\frac{a+b}{c}\right)^3}$; $\sqrt[3]{\left(\frac{x-3}{x+4}\right)^2}$.
7. $3\sqrt{10}$; $5\sqrt{5}$; $3\sqrt[3]{5}$; $9\sqrt{6}$; $18\sqrt[3]{2}$; $\frac{2}{3}\sqrt[3]{12}$; $7\frac{1}{2}$.
8. $8\sqrt[3]{2}$; $6\sqrt[3]{48}$; $2\sqrt[4]{5}$; $2\sqrt[4]{3}$; $10\sqrt{3}$; 2 ; $2\sqrt[3]{18}$;
 12 ; $ab\sqrt[3]{b}$.
9. $\frac{2}{3}\sqrt{2}$; $\frac{2}{3}\sqrt[3]{2}$; $\frac{2}{3}\sqrt[5]{16}$; $ay^{\frac{2}{3}}$; $a\sqrt[3]{a^m}$; $a^2x\sqrt[3]{(ax^2 - 1)}$.
10. $\frac{2}{3}\sqrt[3]{150}$; $\sqrt[4]{375}$; $a^{\frac{1}{2}}(x + 5)$; $(x + y)\sqrt[3]{(x - y)}$.
11. $(x - a)\sqrt[3]{\{(x + a)(x^2 - a^2)\}}$; $x^{\frac{1}{2}}(x + y)$; $2(a - b)\sqrt[3]{(ab)}$.
12. $10\sqrt{3}$, $\frac{7}{2}\sqrt{3}$, $\frac{1}{2}\sqrt{3}$, $\frac{1}{2}\sqrt{3}$, $\frac{1}{2}\sqrt{3}$.
13. $4^{\frac{1}{2}}$, $3^{\frac{1}{2}}$; $8^{\frac{1}{2}}$, $6^{\frac{1}{2}}$; $10,000^{\frac{1}{4}}$, $1000^{\frac{1}{4}}$; $33^{\frac{1}{2}}$, $32^{\frac{1}{2}}$; $80^{\frac{1}{2}}$, $50^{\frac{1}{2}}$;
 $a^{\frac{1}{2}}$, $a^{\frac{2}{3}}$; $a^{\frac{1}{2}}$, $a^{\frac{2}{3}}$; $a^{\frac{1}{2}}$, $a^{\frac{2}{3}}$.
14. $2\sqrt[3]{3} = 24^{\frac{1}{3}}$; $3\sqrt{2} = 18^{\frac{1}{2}}$; $24^{\frac{2}{3}} = 576^{\frac{1}{2}}$; $18^{\frac{2}{3}} = 5832^{\frac{1}{2}}$;
 and $\frac{2}{3}\sqrt[3]{1} = (244\frac{2}{3})^{\frac{1}{2}}$.

EXERCISE XCI. (PAGE 317.)

1. $2\sqrt{2}$; $8\sqrt[3]{5}$. 2. $-12\frac{1}{2}\sqrt{3}$; $11\frac{2}{3}\sqrt[3]{9}$.
3. $60\sqrt{3}$; $80\sqrt{3}$; 24. 4. $6 - 5\sqrt{6}$; $6\sqrt{3} + 3\sqrt{30}$.
5. -32 .
6. $\frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{3} + 2\sqrt{5}$; $\frac{1}{2}\sqrt{6} + \frac{1}{2}\sqrt[3]{32} + \frac{1}{2}\sqrt[4]{120}$.
7. $\frac{2}{3}(\sqrt{7} + \sqrt{3})$; $\frac{1}{2}(7 + 3\sqrt{5})$.
8. $\frac{1}{2}(17 - 3\sqrt{5})$; $\frac{1}{4}(16 - 13\sqrt{2})$;
 $\frac{1}{11}(7\sqrt{14} - 13)$; $2a^2 - 1 + 2a\sqrt{(a^2 - 1)}$.
9. $288\sqrt[3]{72}$; see 4, p. 316.
10. $x + y + z + 2\sqrt{(xy)} - 2\sqrt{(xz)} - 2\sqrt{(yz)}$;
 $13x^2 + 4 + 12x\sqrt{(x^2 + 1)}$.
11. $\sqrt{x - a}$; $x^{\frac{2}{3}} - (xa)^{\frac{1}{3}} - a^{\frac{2}{3}}$; $a - \sqrt{(ab)} + b$;
 $\frac{1}{16}(25 - 6\sqrt{2})(3 - \sqrt[4]{2})$.

12. Square and transpose radicals, square again, then
 $(ax + by + cz)^2 = 4(abxy + bcyz + acxz)$
 $+ 8\sqrt{(abcxyz)}\{\sqrt{(ax)} + \sqrt{(by)} + \sqrt{(bz)}\}$, etc.
13. Rationalize. 14. 3.1003. 15. 3.160.
16. $(\sqrt{5} + 1)\{4 - \sqrt{(10 + 2\sqrt{5})}\} \div 4$; $\{a + \sqrt{(a^2 - x^2)}\} \div x$.
17. $4x\sqrt{(x^2 - 1)}$; $1 \div (1 - x^2)$. 18. $2x^2 \div a^2$.
19. a ; rationalize and substitute.
20. $\sqrt{(a - x)} \div (\sqrt{a} + \sqrt{x})$; factor out $\sqrt{(a + x)}$ in denominator of first fraction and rationalize, resulting numerator cancels denominator of last fraction, etc. 20. $10\frac{1}{2}$.

EXERCISE XCII [a]. (PAGE 320.)

1. $\sqrt{3} + \sqrt{2}$. 2. $\sqrt{11} + \sqrt{2}$. 3. $\sqrt{10} - \sqrt{6}$.
4. $2 + \sqrt{2}$. 5. $\sqrt{11} + \sqrt{5}$. 6. $\sqrt{5} + \sqrt{2}$.
7. $\sqrt{6} + 1$. 8. $2 + \sqrt{5}$. 9. $2\sqrt{3} + 3\sqrt{5}$.
10. $\frac{\sqrt[4]{7}}{\sqrt{2}}(\sqrt{7} - \sqrt{3})$. 11. $\sqrt{7} - \sqrt{3}$.
12. $3 - \sqrt{3}$; change 13 to 11.
13. $2\sqrt{5} - 3$. 14. $3\sqrt{11} - \sqrt{41}$. 15. $\sqrt{7} - \sqrt{2}$.

[b.]

1. $\sqrt[4]{3}(1 + \sqrt{2})$. 2. $\sqrt[4]{5}(1 + \sqrt{2})$. 3. $\sqrt[4]{2}(\sqrt{3} - \sqrt{2})$.
4. $5 + \sqrt{6}$. 5. $\sqrt{51} - 7$. 6. $\sqrt{17} + \sqrt{19}$.
7. $\sqrt[4]{6}(1 + \sqrt{2})$. 8. $\sqrt[4]{2}(\sqrt{3} - \sqrt{2})$. 9. $3\sqrt{5} + 6\sqrt{2}$.
10. $\frac{1}{2}(\sqrt{3} - \sqrt{\frac{3}{2}})$. 11. $\sqrt{30} - \sqrt{\frac{1}{2}}$. 12. $\sqrt[4]{2} + \sqrt[4]{\frac{1}{2}}$.
13. $\sqrt[4]{3}(1 + \sqrt{2})$. 14. $\sqrt{(ab - ab^2)} - \sqrt{(ab^2)}$.
15. $\frac{\sqrt{1+x} + \sqrt{1-x}}{2}$. 16. $3\sqrt{m} - 5\sqrt{n}$.
17. $\sqrt{(m^2 - n^2)} + n$. 18. $\sqrt{(x + y)} + \sqrt{(x - y)}$.
19. $\sqrt{x + y} + \sqrt{z}$. 20. $1 - x + \sqrt{1 + 2x - x^2}$.

21. $1 + \sqrt{2}$. 22. $\sqrt[4]{2}(\sqrt{5} + \sqrt{3})$; in text 60 under root sign.
 23. $\sqrt{5} - 1$. 24. $\sqrt{5} + \frac{1}{2}\sqrt{3}$; 48 for 49 in text.
 25. $\sqrt{3} - \sqrt{2}$. 26. $\frac{1}{2}(\sqrt{10} + \sqrt{2})$.

EXERCISE XCIII [a]. (PAGE 323.)

1. $a^2 \div 2$. 2. 8. 3. $\sqrt{5}$. 4. 21.
 5. 9. 6. 2. 7. a^m . 8. 16.
 9. -1. 10. a . 11. 25, $\frac{1}{25}$, read $5\frac{1}{2}$ in text.
 12. 3. 13. 9. 14. 4 or $-14\frac{2}{3}$. 15. $7\frac{1}{2}$ or 4.
 16. 363. 17. 9. 18. 2. 19. 2.
 20. 5. 21. 6. 22. $3\frac{1}{2}$ or 16. 23. -243 or 32.
 24. $\frac{a+b}{a-b}$. 25. 9. 26. $-\frac{1}{16}$.

[b.]

1. $\frac{1}{b-2}$. 2. $\frac{(a-b)^2}{2a}$. 3. -1. 4. 16.
 5. $-(a+b)$. 6. 46. 7. 6. 8. $\frac{1}{2}$.
 9. $\frac{1}{2}$. In text remove parentheses from denominator.
 10. 27. 11. $\frac{(a-b)^2}{2a-b}$.
 12. $\frac{1}{3}$. 13. ± 3 ; use $\sqrt{x^2+7}$ as the unknown.
 14. 1. 15. -1, 2, 3, 6. 16. $\sqrt[4]{\frac{1}{4}(4a^2b^3-b^4)}$.

[c.] (PAGE 324.)

1. -2, 4, 6, 12.
 2. 0, $\pm \sqrt{3}$; simplifies to $x\{\sqrt{2+x} + \sqrt{2-x}\}$
 $= \sqrt{2}\{2 + \sqrt{4-x^2}\} = \frac{\sqrt{2}}{2}\{4 + 2\sqrt{4-x^2}\}$
 $= \frac{\sqrt{2}}{2}\{\sqrt{2+x} + \sqrt{2-x}\}^2$; $\therefore \sqrt{2+x} + \sqrt{2-x}$
 is a factor, etc.
 3. 0, $(b^2 - 4a^2) \div 4a$.
 4. $a(b-c) \div 2\sqrt{bc}$; see Ex. 5, p. 322.

5. $\frac{1}{a} \left(b - \frac{nc}{n-1} \right)^2$. 6. $0, \pm \sqrt{3}$.
7. $\frac{3}{8}$. 8. $81 \div a$.
9. $2a^2 \div (1 + a^2)$. 10. $-(a^2 + b^2) \div (a + b)$.
11. $(1 + 4b - 10b^2 + 4b^3 + b^4) \div (1 + b)^4$.
12. $\frac{1}{a} \sqrt{\left(\frac{2a}{b} - 1 \right)}$; equation is $\sqrt{\left(\frac{1+bx}{1-bx} \right)} = \frac{1+ax}{1-ax}$;
square and use formula (6), p. 181.
13. 30. 14. $\left\{ \frac{a^2(c-d)^2 - b^2(c+d)^2}{2(c^2+d^2)} \right\}^{\frac{1}{2}}$. 15. $\frac{7}{3}$.
16. $\left\{ \frac{(c^3 - 2a)^3 + 27a^3c^3}{27c^3} \right\}^{\frac{1}{2}}$; cube by formula G (2), p. 85.
17. $\pm \frac{1}{2} \sqrt{\left\{ 1 - \left(\frac{c-2}{3c^{\frac{1}{3}}} \right)^3 \right\}^{\frac{1}{2}}}$.
18. $-\frac{7}{12} \frac{1}{6} \frac{1}{7}, -\frac{10}{12} \frac{2}{3} \frac{3}{3}$; divide by right member, and
 $\left(\frac{1+x}{1-x} \right)^{\frac{1}{6}} + \frac{3}{16} \left(\frac{1-x}{1+x} \right)^{\frac{1}{6}} = 1$, or $y + \frac{3}{16} \cdot \frac{1}{y} = 1$, etc.

EXERCISE XCIV [a]. (PAGE 327.)

1. $-3x^2 + 2x - 1$. 2. $8x^3 - 12x^2 + 6x - 1$.
3. $8x^3 - 12x^2y + 6xy^2 - y^3$. 4. $x^3 - 2x^2y + 2xy^2 - y^3$.
5. $2 - 3x - x^2 + 2x^3$. 6. $x^4 - 2x^3y + 3xy^3 - y^4$.
7. $1 + \frac{1}{3}a - \frac{1}{6}a^2 + \frac{1}{6}a^3 - \frac{5}{128}a^4$.

[b.] (PAGE 328.)

1. $x^2 - x + 1$. 2. $2 - 4x + x^2$.
3. $1 + 3x - x^2$. 4. $y^2 - y + 2$.
5. $x^3 - x^2y + xy^2 + y^3$. 6. $a^2 - \frac{1}{3}a^{-1} + \frac{1}{3}a^{-2}$.
7. $1 + \frac{1}{3}x - \frac{1}{3}x^2 + \frac{5}{81}x^3$.

[c.]

1. $1\frac{3}{4}$. 2. $1\frac{1}{2}$. 3. 10. 4. $-\frac{3}{4}$.
5. $(a^4 - d) \div (c - 2a^3)$. 6. $b \div (a)$.

7. 8. 8. 6. 9. $-\frac{1}{2}$.
10. $\pm 3a$. 11. $(m - n)^2 + a^2 = 0$.
12. Condition for square is $q^2 = 4p^2 \cdot (qr + q^2)$, etc.
13. See Ex. 3, p. 327; remainder in this case is $12ab^2x^2 - 24b^3$,
which must $= 0$, etc.
14. $(a^{\frac{1}{3}}x + \frac{1}{3}a^{-\frac{2}{3}}b)^3 = \text{given expression}$; expand and equate coefficients, $\frac{1}{3}a^{-1}b^2 = c$, $\frac{1}{27}a^{-2}b^3 = d$.









